Bargaining and War: On the Communication Equilibrium in Conflict Games

Youngseok Park and Colin Campbell

The players contemplate whether to take an active engagement action to compel the leader of a neighboring state (an extremist) to give up his risky weapons. We show that a player with greater damage from the extremist is more likely to choose an active engagement action than a player with lesser damage. The likelihood of both players choosing the active engagement action decreases by a hawkish extremist who can send a provocative message, if both players are coordination types. If both players are opportunistic types, a dovish extremist can send an appeasement message that causes one player to be more active while another to be more inactive.
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Executive Summary

We present a version of Baliga and Sjöström’s (2012a) conflict games with two asymmetric players. The players contemplate whether to take an active engagement action to compel the leader of a neighboring state (an extremist) to give up his risky weapons. We show that a player with greater damage from the extremist is more likely to choose an active engagement action than a player with lesser damage. Furthermore, we examine cheap-talk communication equilibria with the extremist. The likelihood of both players choosing the active engagement action decreases by a hawkish extremist who can send a provocative message, if both players are coordination types. If both players are opportunistic types, a dovish extremist can send an appeasement message that causes one player to be more active while another to be more inactive. Lastly, we show that there does not exist any other communication equilibrium for either kind of extremist, for any other combination of player types.

JEL Classification: D74, D82

Keywords: Asymmetry, Cheap-Talk, Conflict Games
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Contents

Executive Summary ........................................... 3

1. Introduction ............................................. 7

2. The Conflict Game with Two Asymmetric Players ............ 10
   2.1 The Conflict Game with Cheap-Talk Communication ........... 12
   2.2 Effective Cheap-Talk Communication .......................... 14
   2.3 Ineffective Communication .................................. 16

3. Discussion ................................................. 18

References ..................................................... 19

Appendix ......................................................... 20
Table

Table 1. The Conflict Game

Figures

Figure 1. Communication-free equilibrium with two coordination types
Figure 2. Communication-free equilibrium with a coordination type and an opportunistic type
Figure 3. Communication equilibrium with two coordination types
Figure 4. Communication equilibrium with two opportunistic types
Figure 5. No communication equilibrium with a coordination type and an opportunistic type (the hawkish extremist)
Figure 6. No communication equilibrium with a coordination type and an opportunistic type (the dovish extremist)
Bargaining and War: On the Communication Equilibrium in Conflict Games

Youngseok Park* and Colin Campbell†

1 Introduction

Multiple states can work jointly toward some common goal in dealing with a risky situation in which a third party (an extremist) is involved. Consider, for example, the United States and China contemplating whether to take an active engagement action to compel the leader of North Korea to give up nuclear weapons: “This whole Six-Party process has done more to bring the United States and China together than any other process I’m aware of,” stated Christopher Hill, the US Assistant Secretary of State and the top negotiator at the Six-Party Talks, after agreement was reached on the Initial Actions for the Implementation of the Joint Statement in Beijing on February 13, 2007 (Glaser and Wang, 2008).

North Korean nuclear weapons may impose asymmetric damages (or threats) to the US and China. Both the US and China may not want North Korea to become a nuclear power, but for different reasons. North Korean nuclear weapons may instill fear to neighboring countries such as South Korea and Japan, and make them forced to be armed with nuclear weapons. But the US may not want South Korea and Japan to be armed with nuclear weapons because it may badly affect regional arms proliferation. On the other hand, China

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may not want North Korea to be a nuclear power because North Korean nuclear weapons may impose obstacles to China’s path to dominance in Asia.\(^1\) Similarly, the benefits of active engagement with North Korea may differ across countries as well. For instance, China has engaged with North Korea more actively than the US because pushing sanctions against North Korea might cause North Korea’s collapse, and China might have to bear a refugee crisis on its border.

To study such an international conflict between two asymmetric states and a third party (an extremist), we present a game-theoretic model which is based on the conflict game of Baliga and Sjöström (2012a, hereafter BS12). BS12 present the conflict game that shows how two symmetric players can be trapped in a state of war (the Hobbesian trap) due to the fear of the counterparty’s hostility. Furthermore, they show how a conflict can be inflamed (mitigated) by an extremist who can send a provocative (peaceful) message.\(^2\)

In the current paper, we consider two asymmetric players, 1 and 2, who are pivotal decision makers of states 1 and 2, respectively. The extremist’s weapons present a greater risk of damage to player 1 than player 2. The two players simultaneously decide to commit to active engagement with the extremist (action \(A\)) or remain inactive (action \(I\)). We investigate (i) how asymmetry comes into play in such a conflict game, and (ii) when the extremist can effectively communicate with the two asymmetric players.\(^3\)

In the communication-free equilibrium (without communication with the extremist), we show that a player with greater damage from the extremist is more likely to choose an active engagement action than a player with lesser damage. We examine cheap-talk communication equilibria with the extremist. The extremist (player \(E\)) sends a publicly observed cheap-talk message before players 1 and 2 make their decisions. The extremist is from (or has close ties with) state 1.\(^4\) The extremist can be either hawkish or dovish, depending on the extremist’s preference. The hawkish extremist wants player 1 to choose \(I\), and the dovish extremist

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\(^1\)The New York Times, 2017b, “North Korea’s Nuclear Arsenal Threatens China’s Path to Power.”

\(^2\)See Baliga and Sjöström (2012b, 2013) for a survey of the conflict theory literature.


\(^4\)For instance, China-North Korea relations have long been described as “lips and teeth” (Revere, 2019).
wants player 1 to choose A. Both kinds of extremist want player 2 to choose A.

In the game with the extremist’s cheap-talk communication, we show two kinds of effective communication equilibrium. First, the likelihood of both players choosing the active engagement action decreases by a hawkish extremist who can send a provocative message, if both players are coordination types. Second, if both players are opportunistic types, a dovish extremist can send an appeasement message that causes one player to be more active while another to be more inactive. Finally, we show that there does not exist any other effective communication equilibrium for either kind of extremist, for any other combination of player types.

Contributions of this paper are two folds. First, we extend BS12’s conflict game with asymmetry, and apply it to the context of joint international political-action game. Second, we investigate the cheap-talk communication further with asymmetric players while BS12 showed only two communication equilibria where both players are coordination types and where both players are opportunistic types. That is, by allowing asymmetry between the players, we complete the proof of the existence of communication equilibrium in conflict games: There does not exist any effective communication equilibrium for either kind of extremist, for any other combination of player types except the two cases.
2 The Conflict Game with Two Asymmetric Players

We consider a version of the conflict game of BS12, with two asymmetric players 1 and 2. Player \( i \in \{1, 2\} \) is the pivotal decision maker of state \( i \). Each player simultaneously chooses either *Active* engagement (action \( A \): talks for denuclearization, economic relief, and etc.) with the leader of a neighboring state or remain *Inactive* (action \( I \)) with him. The leader of the neighboring state carries weapons of massive destruction (WMD), and will be formally introduced to the game later. The players’ payoffs are given by the following matrix, where the row and the column represent the payoffs for players 1 and 2, respectively.

\[
\begin{array}{c|cc}
A & \text{Active} & \text{Inactive} \\
\hline
I & \mu_1 - c_1, \mu_2 - c_2 & \mu_1 - d_1 - c_1, \mu_2 - d_2 \\
\mu_1 - d_1, \mu_2 - d_2 - c_2 & -d_1, -d_2 \\
\end{array}
\]

Table 1: The Conflict Game

Note that \( d_i \) captures the damage to player \( i \) of one of the two players choosing an inactive course of action, which increases the risk of WMD. We assume that the level of damage is asymmetric between the two players, as the damage from the risk of WMD may differ across states. \( \mu_i \) captures the benefit of being active, which arises from active engagement with the neighboring state to curb the risk of WMD. We assume that \( \mu_i \) is asymmetric between the two states. Note that \( \mu_i \) and \( d_i \), for \( i \in \{1, 2\} \), are common knowledge.

Notice that \( c_i \) is player \( i \)’s privately known cost of choosing the active action \( A \), referred to as his type. The players’ types, \( c_1 \) and \( c_2 \), are random variables independently drawn from continuous cumulative distribution functions \( F_1(c_1) \) and \( F_2(c_2) \), respectively, with support \( [\underline{c}, \bar{c}] \), and where \( F_i'(c_i) > 0 \) for all \( c_i \in (\underline{c}, \bar{c}) \) and \( i \in \{1, 2\} \).

Player \( i \) is a **dominant strategy active** if \( A \) is a dominant strategy (\( d_i \geq c_i \) and \( \mu_i \geq c_i \) with at least one strict inequality). Player \( i \) is a **dominant strategy inactive** if \( I \) is a dominant strategy (\( d_i \leq c_i \) and \( \mu_i \leq c_i \) with at least one strict inequality). Player \( i \) is a **coordination type** if \( A \) (\( I \)) is a best response to \( A \) (\( I \)) (\( \mu_i \leq c_i \leq d_i \)). Note that when both players are coordination types, the actions \( A \) and \( I \) have the properties of strategic complements. Player
i is an opportunistic type if I is a best response to A and vice versa \((d_i \leq c_i \leq \mu_i)\). Note that when both players are coordination (opportunistic) types, the actions A and I have the properties of strategic complements (substitutes). Assumption 1 states that the support of \(F_i\) is big enough to include dominant strategy types of both kinds.

**Assumption 1.** \(\bar{c} < \mu_i < \bar{c}\) and \(\bar{c} < d_i < \bar{c}\) for all \(i \in \{1, 2\}\).

Suppose that player \(i\) thinks player \(j\) will choose A with probability \(p_j\). Player i’s expected payoff from choosing A is \(\mu_i - c_i - d_i(1 - p_j)\), while his expected payoff from I is \(\mu_ip_j - d_i\).

Thus, if he chooses A instead of I, his net gain is

\[
\mu_i - c_i + (d_i - \mu_i)p_j. \tag{1}
\]

Player i’s strategy is a function \(\sigma_i : [\bar{c}, \bar{c}] \to \{A, I\}\), which specifies an action \(\sigma_i(c_i) \in \{A, I\}\) for each type \(c_i \in [\bar{c}, \bar{c}]\). In a Bayesian Nash equilibrium (BNE), all types maximize their expected payoffs. Therefore, \(\sigma_i(c_i) = A\) if the expression (1) is positive, and \(\sigma_i(c_i) = I\) if negative. If the expression (1) is zero, then the actions A and I are indifferent. For convenience, we assume that the player chooses A in this case.

Player i uses a cutoff strategy if there is a cutoff point \(x \in [\bar{c}, \bar{c}]\) such that \(\sigma_i(c_i) = A\) if and only if \(c_i \leq x\). Because the expression (1) is monotone in \(c_i\), all BNE must be in cutoff strategies. Therefore, we can restrict our attention to cutoff strategies without loss of generality. Any such strategy can be identified by its cutoff point \(x \in [\bar{c}, \bar{c}]\). As there are dominant strategy types by Assumption 1, all BNE must be interior: each player chooses A with probability strictly between 0 and 1.

If player \(j\) uses cutoff point \(x_j\), the probability that he plays A is \(p_j = F_j(x_j)\). Therefore, using (1), player i’s best response to player j’s cutoff \(x_j\) is to choose the cutoff \(x_i = \Gamma(x_j)\), where

\[
\Gamma_i(x) = \mu_i + (d_i - \mu_i)F_j(x). \tag{2}
\]
The function $\Gamma_i$ is the best-response function for player $i$’s cutoff strategy. The best-response functions generate a unique equilibrium which is ensured by Assumption 2.

**Assumption 2.** $F_i'(c_i) < \frac{1}{d_i - \mu_i}$ for all $c_i \in (\underline{c}, \bar{c})$.

If $F_i$ happens to be uniform, then there is maximal uncertainty (for a given support) and Assumption 2 is redundant. More precisely, with a uniform distribution, $F_i'(c) = \frac{1}{c - \bar{c}}$, so Assumption 1 implies $F_i'(c) < \frac{1}{d_i - \mu_i}$. Note that Assumption 2 is much weaker than uniformity.

**Proposition 1.** The conflict game with asymmetric players has a unique Bayesian Nash equilibrium where the player with the lower value of damage uses the lower cutoff, whether players are coordination or opportunistic types.

*Proof.* See Appendix. \hfill \Box

In the unique BNE, which we refer to as the communication-free equilibrium, player 1 chooses $A$ if $c_1 \leq \bar{y}$, and player 2 chooses $A$ if $c_2 \leq \bar{x}$, where $(\bar{x}, \bar{y})$ is the unique equilibrium point of $\Gamma_1(x)$ and $\Gamma_2(y)$ in $[\underline{c}, \bar{c}]$. The asymmetry of the game implies that each player uses a different cutoff point. More precisely, $\bar{x} < \bar{y}$, which indicates that player 1 is more likely to play $A$ than player 2. Note that the player who uses the lower cutoff in the communication-free equilibrium will be the player with the lower value for $d$. Figures 1 and 2 illustrate the communication-free equilibrium.

### 2.1 The Conflict Game with Cheap-Talk Communication

Now a third player in the conflict game, player $E$ is formally introduced. Player $E$ is the leader of the neighboring country, who is considered to be an extremist. He has close ties with country 1, so he knows player 1’s cost type $c_1$, but not $c_2$. As player $E$ is an extremist, his cost type $c_E$ has an extreme value, which differs from player 1’s. Player $E$’s payoff is obtained by setting $c_1 = c_E$ in the payoff matrix. We assume that $c_E$ is common knowledge.
among the three players.

First, player $E$ can be considered to be a dovish extremist (appeaser) if $c_E < 0$. To put it differently, $(-c_E) > 0$ represents a benefit the dovish extremist enjoys if player 1 chooses $A$. Note that the dovish extremist’s payoffs are higher when player 1 chooses $A$ no matter what player 2 chooses. So the extremist always wants player 1 to choose $A$.

Second, if player $E$ is a hawkish extremist (hardliner), then $c_E > \mu_1 + d_1$. The highest payoff the hawkish extremist can obtain if player 1 chooses $A$ is $\mu_1 - c_E$, while the lowest payoff he can obtain when player 1 chooses $I$ is $-d_1 > \mu_1 - c_E$. Therefore, he always wants player 1 to choose $I$. Notice that, holding player 1’s action fixed, the extremist (dovish or hawkish) is better off if player 2 chooses $A$.

Before players 1 and 2 play the game, player $E$ sends a publicly observed cheap-talk message, $m \in M$, where $M$ is his message space. The time line is as follows.

1. Players 1 and $E$ learn $c_1$. Player 2 learns $c_2$ ($c_E$ is common knowledge).
2. Player $E$ sends a publicly observed cheap-talk message $m \in M$.
3. Players 1 and 2 simultaneously choose $A$ or $I$.

A strategy of player $E$ is a function $m : [\underline{c}, \bar{c}] \rightarrow M$, where $m(c_1)$ is the message sent by player $E$ when player 1’s type is $c_1$. Without loss of generality, we assume that each player $j \in \{1, 2\}$ uses a conditional cutoff strategy: for any message $m \in M$, there is a cutoff $c_j(m)$ such that if player $j$ hears message $m$, then he chooses $A$ if and only if $c_j \leq c_j(m)$. Lemma 1 states Baliga and Sjöström’s (2012a) result for the communication equilibrium.

**Lemma 1.** In the communication equilibrium, we can assume without loss of generality that $M$ contains only two messages, $M = \{m_0, m_1\}$, where $c_2(m_1) > c_2(m_0)$.

**Proof.** See Appendix. \hfill $\square$
2.2 Effective Cheap-Talk Communication

Here, we present the communication equilibria following the results of Baliga and Sjöström (2012a). First, we consider the case where both players are coordination types, \( d_i > \mu_i \) for all \( i \in \{1, 2\} \). Suppose player \( E \) is a hawkish extremist, \( c_E > \mu_1 + d_1 \), and both players are coordination types. There exists a communication equilibrium, where the hawkish extremist uses cheap-talk to decrease the likelihood of cooperation for active engagement below the level where it would be in the communication-free equilibrium.

**Proposition 2.** Suppose that player \( E \) is a hawkish extremist and both players 1 and 2 are coordination types. A communication equilibrium exists. All types of players 1 and 2 prefer the communication-free equilibrium to any communication equilibrium. Player \( E \) is better off in the communication equilibrium if and only if \( \Gamma_1(\mu_2) < c_1 < \overline{y} \). If (8) holds for all \( y \in (\underline{y}, \overline{y}) \), then there is a unique communication equilibrium.

*Proof.* See Appendix. \( \square \)

In the communication-free equilibrium, the probability of active engagement cooperation, in the sense that the outcome is \( AA \), is \( F_2(\overline{x})F_1(\overline{y}) \). In the communication equilibrium, \( AA \) happens with probability \( \Gamma_1(\mu_2)F_2(x^*) < F_2(\overline{x})F_1(\overline{y}) \). Thus, active engagement cooperation is less likely in the communication equilibrium than in the communication-free equilibrium.

To understand how the cutoff points can be uniformly higher with cheap-talk, we interpret message \( m_1 \) as being “no provocation” and message \( m_0 \) as being “provocation”. A spiral of hawkishness occurs when player 1 is a coordination type, \( c_1 \in (\Gamma_1(\mu_2), \overline{y}) \), who would have played \( A \) in the communication-free equilibrium. Now he plays \( I \) instead, and so does player 2 (except if he is a dominant strategy active). The players choose inactive actions following provocation \( (m_0) \) because they think the other will choose an inactive action. The fact that a provocation does not show also deters cooperation.\(^5\) Thus, the communication equilibrium

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\(^5\)Baliga and Sjöström (2012a) explains the logic behind this as follows: “In the “curious incident of the dog in the night-time” from Sir Arthur Conan Doyle’s short story “Silver Blaze,” the dog did not bark at an intruder because the dog knew him well.”

14 Bargaining and War: On the Communication Equilibrium in Conflict Games
has less active engagement cooperation than the communication-free equilibrium.

Next, we consider the case where both players are opportunistic types, \( d_i < \mu_i \) for all \( i \in \{1, 2\} \), and player \( E \) is a dovish extremist, \( c_E < 0 \). There exists a communication equilibrium where the dovish extremist sends informative messages. Now we interpret message \( m_0 \) as “appeasement” and message \( m_1 \) as “no appeasement.” Note that the appeasement message \( (m_0) \) will make player 2 more inactive, and player 1 more active and chooses \( A \). We present the following proposition.

**Proposition 3.** Suppose that player \( E \) is a dovish extremist and both players are opportunistic types. A communication equilibrium exists. All of player 1’s types prefer the communication-free equilibrium to the communication equilibrium. All of player 2’s types have the opposite preference. Player \( E \) is better off in the communication equilibrium if and only if \( y^* < c_1 < \Gamma_1(d_2) \). If (11) holds for all \( y \in (\underline{c}, \bar{c}) \) then there is a unique communication equilibrium.

**Proof.** See Appendix.

What impact does the appeasement message have on the probability of cooperation on active engagement? In the communication-free equilibrium, \( \bar{x} \) and \( \bar{y} \) are the cutoffs of players 2 and 1, respectively. Note that player 2’s cutoff points \( x^* \) and \( d_2 \) are strictly lower than \( \bar{x} \). Thus, any communication makes player 2 more conservative, no matter what message is actually sent. On the other hand, player 1’s cutoff points \( y^* \) and \( \Gamma_1(d_2) \) are strictly higher than \( \bar{y} \). Thus, communication makes player 1 more progressive, no matter what message is actually sent. Since one player becomes more progressive and the other less, it is not possible to unambiguously say if communication is good or bad for cooperation on active engagement.

The welfare effects are unambiguous. As player 1 is more likely to play \( A \) in the communication equilibrium, player 2 is made better off. Conversely, as player 2 is more likely to play \( I \), player 1 is made worse off. The dovish extremist is made better off by the dovish message,
because they prevent player 1 from choosing I. On the other hand, the “dog that did not bark” effect makes player 2 more likely to choose I when there is the “no appeasement” message, and this makes player E worse off.

2.3 Ineffective Communication

Now we examine if there exists any possible communication equilibrium with any other combination of player types. In a communication equilibrium, let $m_1$ be the message that induces player 2 to play A for a larger set of realizations of $c_2$, and $m_0$ to be the message that induces player 2 to play A for a smaller set of realizations of $c_2$. Given $c_E$, let $a_1^E$ be the generic action that an extremist always prefers player 1 to choose (only the cases in which this preference is constant across $a_2^E$ are considered). Let $a_1^{-E}$ be player 1’s other action.

Since the extremist always prefers that player 2 plays A, a necessary condition for equilibrium is that for all types $c_1$ such that an extremist sends $m_0$, it must be the case that all such types play $a_1^E$ when $m_0$ is sent, but would play $a_1^{-E}$ if $m_1$ were sent. However, since message $m_0$ being sent guarantees that player 1 will play $a_1^E$ in equilibrium, this is only consistent with $m_0$ resulting in player 2 choosing I more often if I is player 2’s best response to $a_1^E$. In particular, after $m_0$, player 2 chooses I for all realizations of $c_2$ except for those such that A is dominant for player 2. Given player 2 choosing I, player 1 having a type such that playing $a_1^E$ is optimal after $m_0$, but $a_1^{-E}$ is optimal after $m_1$, is possible only if $a_1^E$ is player 1’s best response when player 2 plays I, but $a_1^{-E}$ is player 1’s best response when player 2 plays A.

To summarize, we need: for $a_1^E$ to be player E’s preferred action for player 1 when player 1 does not have a dominant action, $a_1^E$ is player 1’s best response when player 2 chooses I; and when player 1 chooses $a_1^E$, player 2’s best response is I, except when A is dominant for player 2. Thus, since $a_1^E \in \{A, I\}$, there are only two possibilities for a communication equilibrium: First, player E prefers that player 1 chooses A (namely, player E is the dovish extremist); A is player 1’s best response when player 2 chooses I; I is player 2’s best response
when player 1 chooses $A$. Second, player $E$ prefers that player 1 chooses $I$ (namely, player $E$ is the hawkish extremist); $I$ is player 1’s best response when player 2 chooses $I$; $I$ is player 2’s best response when player 1 chooses $I$.

The results are summarized in the following proposition.

**Proposition 4.** *There does not exist any effective communication equilibrium for either kind of extremist, for any other combination of player types.*

*Proof.* See Appendix. \[\square\]
3 Discussion

If the US and China’s political action game has the property of strategic complements, North Korea can send a hawkish message (e.g., making a hawkish statement or showing a movement at the nuclear facility) that would make both states less likely to choose an active engagement action. If the game has the property of strategic substitutes, North Korea can send a dovish message (e.g., sending a kind letter to the president of the US or meeting the president of South Korea) that would make one state more likely to choose an active action while another less likely. However, North Korea’s strategic communication (cheap-talk) cannot be effective except the two cases.

For future research, we suggest to extend the game by allowing incomplete information in other parameters, such as benefits from being active or damage from risky weapons.


______. 2017b. “North Korea’s Nuclear Arsenal Threatens China’s Path to Power.” (September 5)
Appendix

Proof of Proposition 1. Equilibria must be in cutoff strategies, and must be interior by Assumption 1. The best-response function $\Gamma_i$, defined by (2), is continuous, with $\Gamma_i(\zeta) = \mu_i > \zeta$ and $\Gamma_i(\bar{\zeta}) = d_i < \bar{\zeta}$; therefore it has an equilibrium point $(\bar{x}, \bar{y})$ where $\bar{x} \in [\zeta, \bar{\zeta}]$ and $\bar{y} \in [\zeta, \bar{\zeta}]$ are the cutoff point of player 2 and player 1, respectively. Note that $\bar{x} < \bar{y}$ when $d_1 > d_2$. If players 1 and 2 use cutoffs $\bar{y}$ and $\bar{x}$, respectively, the strategies form a BNE. It remains to show this BNE is unique. Notice that $\Gamma_i'(x) = (d_i - \mu_i)F'(x)$, so the best-response function is upward (downward) sloping if actions are strategic complements (substitutes). In either case, a well-known sufficient condition for uniqueness is that best-response functions have slope strictly less than one in absolute value. Assumption 2 implies that $0 < \Gamma_i'(x) < 1$ if $d_i > \mu_i$, and $-1 < \Gamma_i < 0$ if $d_i < \mu_i$. Hence, the best-response functions cross at most once and there is a unique equilibrium. The sufficient condition for uniqueness of the equilibrium also holds even if the players are of different types. Namely, if player 1 is a coordination type ($\mu_1 < d_1$) and player 2 is an opportunistic type ($d_2 < \mu_2$), the best-response functions have slope strictly less than one in absolute value as long as Assumption 2 holds. 

Proof of Lemma 1. Suppose strategy $\mu_2$ is a part of a BNE. Because unused messages can simply be dropped, we may assume that for any $m \in M$, there is $c_1$ such that $m(c_1) = m$. Now consider any two messages $m$ and $m'$. If $c_2(m) = c_2(m')$, then the probability player 2 plays $A$ is the same after $m$ and $m'$, and this means that each type of player 1 also behaves the same after $m$ as after $m'$. If all players behave the same after $m$ and $m'$, having two separate messages $m$ and $m'$ is redundant. Hence, without loss of generality, we assume $c_2(m) \neq c_2(m')$ whenever $m \neq m'$. Whenever player 1 is a dominant strategy type, player $E$ will send whatever message maximizes the probability that player 2 plays $A$. Call this message $m_1$. Thus,

$$m_1 = \arg \max_{m \in M} c_2(m).$$

(3)

Message $m_1$ is the unique maximizer of $c_2(m)$, since $c_2(m) \neq c_2(m')$ whenever $m \neq m'$. 

20 Bargaining and War: On the Communication Equilibrium in Conflict Games
Player $E$ cannot always send $m_1$, because then messages would not be informative and cheap-talk would be ineffective (contradicting the definition of a communication equilibrium). However, since message $m_1$ uniquely maximizes the probability that player 2 chooses $A$, player $E$ must have some other reason for choosing $m(c_1) \neq m_1$. Specifically, if player $E$ is a dovish extremist (who wants player 1 to choose $A$), then it must be that type $c_1$ would choose $I$ following $m_1$ but $A$ following $m(c_1)$; conversely if player $E$ is a hawkish extremist (who wants player 1 to choose $I$), then it must be that type $c_1$ would choose $A$ following $m_1$ but $I$ following $m(c_1)$. This is the only way that player $E$ can justify sending any other message than $m_1$. Thus, if player $E$ is a dovish extremist, then whenever he sends a message $m_0 \neq m_1$, player 1 will play $A$. Player 2 therefore responds with $A$ whenever $c_2 < d_2$. That is, $c_2(m_0) = d_2$. Since $c_2(m) \neq c_2(m')$ whenever $m \neq m'$, $m_0$ is unique. Thus, $M = \{m_0, m_1\}$.

Similarly, if player $E$ is a hawkish extremist, then whenever he sends a message $m_0 \neq m_1$, player 1 will play $I$. Player 2’s cutoff point must therefore be $c_2(m_0) = \mu_2$. Again, this means $M = \{m_0, m_1\}$. □

**Proof of Proposition 2.** First, it is commonly known that player $E$ wants player 2 to choose $A$ and player 1 to choose $I$. Recall that $M = \{m_0, m_1\}$ by Lemma 1, where $c_2(m_1) > c_2(m_0)$, and to interpret message $m_0$ as a “provocation” and message $m_1$ as “no provocation.” Player 1 is a susceptible type if he chooses $I$ following the message $m_0$, but $A$ following $m_1$. The set of susceptible types is $S = (c_1(m_0), c_1(m_1))$. The proof of Lemma 1 showed that if $m(c_1) = m_0$, then type $c_1$ must be susceptible. Since the hawkish message makes player 2 more likely to choose $I$, player $E$ will only send $m_0$ if it causes player 1 to change his action from $A$ to $I$. On the other hand, player $E$ wants player 1 to choose $I$ and therefore strictly prefer to send $m_0$ whenever player 1 is susceptible. That is, it is optimal for player $E$ to set $m(c_1) = m_0$ if and only if $c_1 \in S$. Accordingly, message $m_0$ signals that player 1 will choose $I$. As argued in the proof of Lemma 1, this implies that $c_2(m_0) = \mu_2$.

Therefore, if $m_0$ is sent then player 2 will choose $A$ with probability $F_2(\mu_2)$, so player 1
prefers A if and only if
\[-c_1 + (1 - F_2(\mu_2))\mu_1 \geq F_2(\mu_2)(-d_1),\]
which is equivalent to \(c_1 \leq \Gamma_1(\mu_2)\). Thus, player 1 uses cutoff point \(c_1(m_0) = \Gamma_1(\mu_2)\), where \(\Gamma_1\) is defined by (2).

It remains only to consider how players 1 and 2 behave when player E shows no provocation (message \(m_1\)). Let \(y^* = c_1(m_1)\) and \(x^* = c_2(m_1)\) denote the cutoff points in this case. Therefore, if \(m_1\) is sent then player 2 will choose A with probability \(F_2(x^*)\), so player 1 prefers A if and only if
\[-c_1 + (1 - F_2(x^*))\mu_1 \geq F_2(x^*)(-d_1),\]
which is equivalent to \(c_1 \leq \Gamma_1(x^*)\). Thus, \(y^* = \Gamma_1(x^*)\). When player 2 hears message \(m_1\), he knows that player 1 is not the susceptible type. That is, \(c_1\) is either below \(\Gamma_1(\mu_2)\) or above \(y^*\), and player 1 chooses A in the former case and I in the latter case. Therefore, player 2 prefers A if and only if
\[-c_2 + \frac{1 - F_1(y^*)}{1 - F_1(y^*) + F_1(\Gamma_1(\mu_2))}\mu_2 \geq \frac{1 - F_1(\Gamma_1(\mu_2))}{1 - F_1(y^*) + F_1(\Gamma_1(\mu_2))}(-d_2).\]  

(4)

Inequality (3) is equivalent to \(c_2 \leq \Omega_2(y^*)\), where
\[\Omega_2(\gamma) = \frac{[1 - F_1(\gamma)]\mu_2 + F_1(\Gamma_1(\mu_2))d_2}{[1 - F_1(\gamma)] + F_1(\Gamma_1(\mu_2))},\]
Thus, \(x^* = \Omega_2(y^*)\).

To summarize, any communication equilibrium must have the following form. Player E sets \(m(c_1) = m_0\) if and only if \(c_1 \in S = (\Gamma_1(\mu_2), y^*)\). Player 1's cutoff points are \(c_1(m_0) = \Gamma_1(\mu_2)\) and \(c_1(m_1) = y^*\). Player 2's cutoff points are \(c_2(m_0) = \mu_2\) and \(c_2(m_1) = x^*\).
Moreover, \( x^* \) and \( y^* \) must satisfy \( y^* = \Gamma_1(x^*) \) and \( x^* = \Omega_2(y^*) \). Conversely, if such \( x^* \) and \( y^* \) exist, then they define a communication equilibrium. Figure 3 shows a graphical illustration of a communication equilibrium.

By Assumption 2, \( \Gamma_i \) is increasing with a slope less than one. Since \( F_i(\bar{c}) = 0 \) and \( F_i(\bar{c}) = \mu_i > \bar{c} \) and \( \Gamma_i(\bar{c}) = d_i < \bar{c} \). Furthermore,

\[
\Gamma_i(d_i) - \mu_i = F_j(d_i)(d_i - \mu_i) < d_i - \mu_i.
\]

Therefore,

\[
\Gamma_i(d_i) < d_i. \quad (5)
\]

Also,

\[
\Gamma_i(\mu_j) = \mu_i(1 - F_j(\mu_j)) + d_iF_j(\mu_j) > \mu_i, \quad (6)
\]
as \( d_i > \mu_i \). Let \((\bar{x}, \bar{y})\) be the unique communication-free equilibrium point in \([c, \bar{c}]\). Clearly, \( \mu_2 < \Gamma_2(\mu_1) < \bar{x} \) and \( \mu_1 < \Gamma_1(\mu_2) < \bar{y} \) (see Figure 3).

Notice that

\[
\Omega_2'(y) = \frac{F'_1(y)(d_2 - \mu_2)F_1(\Gamma_1(\mu_2))}{\{[1 - F_1(y)] + F_1(\Gamma_1(\mu_2))\}^2},
\]
so \( \Omega_2 \) is increasing. It is easy to check that \( \Gamma_2(y) > \Omega_2(y) \) whenever \( y \in (\Gamma_1(\mu_2), \bar{c}) \).

Moreover,

\[
\Omega_2(\bar{c}) = \Gamma_2(\bar{c}) = d_2
\]
and

\[
\Omega_2(\Gamma_1(\mu_2)) = \Gamma_2(\Gamma_1(\mu_2)) > \Gamma_1(\mu_2),
\]
where the inequality follows from (5) and the fact that \( \Gamma_2 \) is increasing. These properties are drawn in Figure 3. Notice that the curve \( x = \Omega_2(y) \) lies to the left of the curve \( x = \Gamma_2(y) \) for all \( y \in (\Gamma_1(\mu_2), \bar{c}) \), but that the two curves intersect when \( y = \Gamma_1(\mu_2) \) and \( y = \bar{c} \).

As shown in Figure 3, the two curves \( x = \Omega_2(y) \) and \( y = \Gamma_1(x) \) must intersect at some
\[(x^*, y^*)\), and it must be true that

\[\mu_1 < \Gamma_1(\mu_2) < x^* < y^* < \bar{y}.\] (7)

By construction, \(y^* = \Gamma_1(x^*)\) and \(x^* = \Omega_2(y^*)\). Thus, a communication equilibrium exists. Both players 1 and 2 are strictly more likely to choose \(I\) in a communication equilibrium than in the communication-free equilibrium. To see this illustrated, notice that in the communication-free equilibrium, player 1’s cutoff is \(\bar{y}\) and player 2’s cutoff is \(\bar{x}\). By (6), the cutoff points are strictly lower in the communication equilibrium; namely, \(x^* < \bar{x}\) and \(y^* < \bar{y}\). Thus, whenever a player would have chosen \(I\) in the communication-free equilibrium, he necessarily chooses \(I\) in the communication equilibrium. Moreover, after any message, there are types (of each player) who choose \(I\), but who would have chosen \(A\) in the communication-free equilibrium. It follows that all types of players 1 and 2 are made worse off by communication, because each wants the other player to choose \(A\).

For player \(E\), the welfare comparison across equilibria is ambiguous, because cheap-talk makes both players 1 and 2 more likely to choose \(I\). There are three specific cases. First, if either \(c_1 \leq \Gamma_1(\mu_2)\) or \(c_1 > \bar{y}\), then player 1’s action is the same in the communication equilibrium and in the communication-free equilibrium, but player 2 is more likely to choose \(I\) in the former, making player \(E\) worse off. Second, if \(y^* < c_1 < \bar{y}\), then player 1 would have chosen \(A\) in the communication-free equilibrium. In the communication equilibrium, there is the provocation message when \(y^* < c_1 < \bar{y}\), but player 1 plays \(I\) rather than \(A\), because player 2 is more likely to choose \(I\). Third, if \(\Gamma_1(\mu_2) < c_1 \leq y^*\), then the provocation message causes player 1 to play \(I\), rather than \(A\) in the communication-free equilibrium. Player \(E\) gets a strictly higher payoff when player 1 chooses \(I\) no matter what player 2 chooses. Thus, player \(E\) is better off if player 1 switches to \(I\).

The communication equilibrium is unique if the two curves \(x = \Omega_2(y)\) and \(y = \Gamma_1(x)\) have a unique intersection. This would be true, for example, if \(F\) were concave, because in this case
both $\Omega_2$ and $\Gamma_1$ would be concave. However, uniqueness also obtains without concavity, if a “conditional” version of Assumption 2 holds. Intuitively, after $m_1$ is sent player 2 knows that player 1’s type is either below $\Gamma_1(\mu_2)$ or above $y^*$. Thus, the continuation equilibrium must be the equilibrium of a “conditional” game (without communication) where it is commonly known that player 1’s type distribution has support $[\underline{c}, \Gamma_1(\mu_2)] \cup (y^*, \bar{c}]$ and density

$$g(c) \equiv \frac{F_1'(c)}{1 - F_1(y^*) + F_1(\Gamma_1(\mu_2))}$$
on this support. Furthermore, following $m_1$, player 1’s type $y^*$ must be indifferent between choosing $A$ and $I$. That is, in the “conditional” game, the cutoff type is $y^*$. Recall that Assumption 2 guarantees uniqueness in the “unconditional” communication-free game. The analogous condition which guarantees uniqueness in the “conditional” game is $g(y^*) < 1/(d_i - y)$. Thus, the “conditional” game has a unique equilibrium if the following “conditional” version of Assumption 2 holds:

$$\frac{F_1'(y)}{1 - F_1(y) + F_1(\Gamma_1(\mu_2))} < \frac{1}{d_i - \mu_i} \quad (8)$$

for all $y \in (\underline{c}, \bar{c})$. This implies, since $0 < \Gamma_1'(x) < 1$, that the two curves intersect only once, as indicated in Figure 2. Thus, as before, the requirement for uniqueness is that the distribution is sufficiently diffuse. \hfill \Box

**Proof of Proposition 3.** Player 1 is a susceptible type if his action depends on which message is sent. A susceptible type switches from $I$ to $A$ when they hear message $m_0$. That is, the set of susceptible types is

$$S \equiv (c_1(m_1), c_1(m_0)].$$

The proof of Lemma 1 showed that if $m(c_1) = m_0$, then type $c_1$ must be susceptible. Since the appeasement message ($m_0$) makes player 2 more likely to choose $I$, player $E$ would
not engage in the appeasement message unless player 1 is a susceptible type. Conversely, whenever player 1 is a susceptible type, the dovish extremist will engage in the appeasement message, since he wants player 1 to choose A. Therefore, \( m(c_1) = m_0 \) if and only if \( c_1 \in S \). Accordingly, message \( m_0 \) signals that player 1 will choose A. As argued in the proof of Lemma 1, this implies that \( c_2(m_0) = d_2 \), and player 1’s best response to this cutoff point is \( c_1(m_0) = \Gamma_1(d_2) \).

It remains only to consider how players 1 and 2 behave when the message is “no appeasement” (\( m_1 \)). Let \( y^* = c_1(m_1) \) and \( x^* = c_2(m_1) \) denote the cutoff points used in this case. Arguing as for the case where both players are coordination types, the cutoff points must satisfy \( y^* = \Gamma_1(x^*) \) and \( x^* = \tilde{\Omega}_2(y^*) \), where

\[
\tilde{\Omega}_2(y) = \frac{[1 - F_1(\Gamma_1(d_2))]\mu_2 + F_1(y)d_2}{[1 - F_1(\Gamma_1(d_2))] + F_1(y)}.
\]

As shown in Figure 4, \((x^*, y^*)\) is an intersection of the two curves \( x = \tilde{\Omega}_2(y) \) and \( y = \Gamma_1(x) \). In the case where both players are coordination types, Assumption 2 implies

\[-1 < \Gamma_1'(x) < 0.\]

Furthermore, \( \Gamma_1(\bar{c}) = \mu_1 < \bar{c} \) and \( \Gamma_1(\underline{c}) = d_1 > \underline{c} \), and

\[
\Gamma_1(d_2) - d_1 = (1 - F_2(d_2))(\mu_1 - d_1)
\]

where

\[0 < (1 - F_2(d_2))(\mu_1 - d_1) < \mu_1 - d_1.\]

Therefore,

\[d_1 < \Gamma_1(d_2) < \mu_1.\]  \quad (9)

Let \((\overline{x}, \overline{y})\) be the unique communication-free equilibrium in \([\underline{c}, \bar{c}]\). Clearly, \( d_1 < \overline{x} < \overline{y} < \mu_1 \)

Bargaining and War: On the Communication Equilibrium in Conflict Games
Appendix 27

Figure 4 shows three curves: \( x = \tilde{\Omega}_2(y) \), \( y = \Gamma_1(x) \), and \( x = \Gamma_2(y) \). The curves \( y = \Gamma_1(x) \) and \( x = \Gamma_2(y) \) intersect at the unique communication-free equilibrium, \((\tilde{x}, \tilde{y})\). It is easy to check that \( \Gamma_2(y) > \tilde{\Omega}_2(y) \) whenever \( y \in (\underline{c}, \Gamma_1(d_B)) \). Moreover,

\[
\tilde{\Omega}_2(\underline{c}) = \Gamma_2(\underline{c}) = \mu_2
\]

and

\[
\tilde{\Omega}_2(\Gamma_1(d_2)) = \Gamma_2(\Gamma_A(d_2)) < \Gamma_1(d_2),
\]

where the inequality follows from the fact that \( \Gamma_2 \) is decreasing. Consider now \((x^*, y^*)\) such that \( y^* = \Gamma_1(x^*) \) and \( x^* = \tilde{\Omega}_2(y^*) \), i.e., the intersection of the two curves \( y = \Gamma_1(x) \) and \( x = \tilde{\Omega}_2(y) \). Figure 4 reveals that there exists \((x^*, y^*) \in [\underline{c}, \bar{c}]^2\) such that \( y^* = \Gamma_1(x^*) \) and \( x^* = \tilde{\Omega}_B(y^*) \), and

\[
d_2 < x^* < \bar{x} < y^* < \Gamma_1(d_2) < \mu_1.
\]

Thus, a communication equilibrium exists. Finally, consider whether the communication equilibrium is unique. Using the same argument as before, we must impose a “conditional” version of Assumption 2. Specifically,

\[
\frac{F_1'(c)}{1 - F_1(\Gamma_1(d_2)) + F_1(y)} < \frac{1}{\mu_i - d_i}
\]

for all \( y \in (\underline{c}, \bar{c}) \). It can be checked that (11) implies \(-1 < \tilde{\Omega}_2'(y) < 0\). In this case, since \(-1 < \Gamma_1'(x) < 0\), the two curves intersect only once, as indicated in Figure 4.

\[\Box\]

**Proof of Proposition 4.** First, we show that if player \( E \) is a dovish extremist, \( c_E < 0 \), then he cannot communicate effectively when actions are strategic complements. From Lemma 1, \( M = \{m_0, m_1\} \) with \( c_2(m_1) > c_2(m_0) \). Thus, player 2 is more likely to choose \( A \) after \( m_1 \) than after \( m_0 \). The dovish extremist wants both players 1 and 2 to play \( A \), so he would only choose \( m_0 \) if such a message causes player 1 to play \( A \). Formally, if \( m(c_1) = m_0 \), then
we must have \( c_1 < c_1(m_0) \), so that type \( c_1 \) chooses \( A \) when he hears message \( m_0 \). But if \( c_1 < c_1(m_0) \) for all \( c_1 \) such that \( m(c_1) = m_0 \), then player 2 expects player 1 to play \( A \) for sure when player 2 hears \( m_0 \), so player 2’s cutoff point must be \( c_2(m_0) = d_2 \). However, with \( d_2 > \mu_2 \), types above \( d_2 \) are dominant strategy types who always play \( I \), so it is a contradiction for \( c_2(m_1) > d_2 \). Thus, if player \( E \) is a dovish extremist and the game has strategic complements \( (d_i > \mu_i) \), then cheap-talk cannot be effective. \( \square \)

Next, we show that a communication equilibrium does not exist if player \( E \) is a hawkish extremist, player 1 has strategic complements \( (\mu_1 < d_1) \), and player 2 has strategic substitutes \( (d_2 < \mu_2) \). Any communication equilibrium must have the following form. Player \( E \) sets \( m(c_1) = m_0 \) if and only if \( c_1 \in S = (\Gamma_1(\mu_2), y^\ast] \). Player 1’s cutoff points are \( c_1(m_0) = \Gamma_1(\mu_2) \) and \( c_1(m_1) = y^\ast \). Player 2’s cutoff points are \( c_2(m_0) = \mu_2 \) and \( c_2(m_1) = x^\ast \). Moreover, \( x^\ast \) and \( y^\ast \) must satisfy \( y^\ast = \Gamma_1(x^\ast) \) and \( x^\ast = \Omega_2(y^\ast) \). We show that such \( x^\ast \) and \( y^\ast \) do not exist.

By Assumption 2, \( \Gamma_1 \) is increasing and \( \Gamma_2 \) is decreasing with a slope less than one. Since \( F(\underline{c}) = 0 \) and \( F(\bar{c}) = 1 \), \( \Gamma_1(\underline{c}) = \mu_1 \geq \underline{c}, \Gamma_1(\bar{c}) = d_1 \leq \bar{c}, \Gamma_2(\underline{c}) = \mu_2 \leq \underline{c}, \) and \( \Gamma_2(\bar{c}) = d_1 \geq \bar{c} \). Furthermore,

\[
\Gamma_1(d_1) - \mu_1 = F_2(d_1)(d_1 - \mu_1) < d_1 - \mu_1,
\]

\[
\Gamma_2(d_2) - \mu_2 = F_1(d_2)(d_2 - \mu_2) > d_2 - \mu_2.
\]

Therefore, \( \Gamma_1(d_1) < d_1 \) and \( \Gamma_2(d_2) > d_2 \). Also,

\[
\Gamma_1(\mu_2) = \mu_1(1 - F_2(\mu_2)) + d_1 F_2(\mu_2) > \mu_1, \tag{12}
\]

\[
\Gamma_2(\mu_1) = \mu_2(1 - F_1(\mu_1)) + d_2 F_1(\mu_1) < \mu_2. \tag{13}
\]

Let \( (\bar{\tau}, \bar{y}) \) be the unique communication-free equilibrium point in \([\underline{c}, \bar{c}]\). Clearly, \( \bar{\tau} < \Gamma_2(\mu_1) < \mu_2 \) and \( \mu_1 < \bar{y} < \Gamma_1(\mu_2) \) (see Figure 5).

Notice that

\[
\Omega_2(y) = \frac{F_1(y)(d_2 - \mu_2)F_1(\Gamma_1(\mu_2))}{\{[1 - F_1(y)] + F_1(\Gamma_1(\mu_2))\}^2},
\]

28 Bargaining and War: On the Communication Equilibrium in Conflict Games
so $\Omega_2$ is decreasing. It is easy to check that $\Gamma_2(y) < \Omega_2(y)$ whenever $y \in (\Gamma_1(\mu_2), \bar{c})$.

Moreover, 

$$\Omega_2(\bar{c}) = \Gamma_2(\bar{c}) = d_2$$

and 

$$\Omega_2(\Gamma_1(\mu_2)) = \Gamma_2(\Gamma_1(\mu_2)) < \Gamma_1(\mu_2)$$

where the inequality follows from (6) and the fact that $\Gamma_2$ is decreasing. Notice that the curve $x = \Omega_2(y)$ lies to the right of the curve $x = \Gamma_2(y)$ for all $y \in (\Gamma_1(\mu_2), \bar{c})$, but that the two curves intersect when $y = \Gamma_1(\mu_2)$ and $y = \bar{c}$ (see Figure 5). Thus, a communication equilibrium does not exist.

Next, we show that a hawkish extremist cannot communicate effectively when actions are strategic substitutes. From Lemma, $M = \{m_0, m_1\}$ with $c_2(m_1) > c_2(m_0)$. Thus, player 2 is more likely to choose $A$ after $m_1$ than after $m_0$. The hawkish extremist wants player 1 (but not player 2) to play $I$, so he would only choose $m_0$ if this message causes player 1 to play $I$. But if player 1 plays $I$ for sure after $m_0$, player 2’s cutoff point must be $c_2(m_0) = \mu_2$. However, with $d_2 < \mu_2$, types above $\mu_2$ are dominant strategy types who always play $I$, so it is a contradiction for $c_2(m_1) > \mu_2$. Thus, if player $E$ is a hawkish extremist and the game has strategic substitutes ($d_i < \mu_i$), then cheap-talk cannot be effective.

Finally, we show that a communication equilibrium does not exist if player $E$ is a dovish extremist, player 1 has strategic complements ($\mu_1 < d_1$), and player 2 has strategic substitutes ($d_2 < \mu_2$). In a communication equilibrium, the cutoff points must satisfy $y^* = \Gamma_1(x^*)$ and $x^* = \tilde{\Omega}_2(y^*)$, where

$$\tilde{\Omega}_2(y) = \frac{[1 - F_1(\Gamma_1(d_2))]\mu_2 + F_1(y)d_2}{[1 - F_1(\Gamma_1(d_2)) + F_1(y)].}$$

$(x^*, y^*)$ must be an intersection of the two curves $x = \tilde{\Omega}_2(y)$ and $y = \Gamma_1(x)$. With
strategic substitutes, Assumption 2 implies

\[ 0 < \Gamma'_1(x) < 1. \]

Furthermore, \( \Gamma_1(\xi) = \mu_1 > \xi \) and \( \Gamma_1(\bar{c}) = d_1 < \bar{c} \), and

\[ \Gamma_1(d_2) - d_1 = (1 - F_2(d_2))(\mu_1 - d_1) \]

where

\[ \mu_1 - d_1 < (1 - F_2(d_2))(\mu_1 - d_1) < 0. \]

Therefore,

\[ \mu_1 < \Gamma_1(d_2) < d_1. \tag{14} \]

Let \((\bar{x}, \bar{y})\) be the unique communication-free equilibrium in \([c, \bar{c}]\). Clearly, \( \mu_1 < \bar{x} < \bar{y} < d_1 \) (see Figure 6).

Figure 6 shows three curves: \( x = \tilde{\Omega}_2(y) \), \( y = \Gamma_1(x) \), and \( x = \Gamma_2(y) \). The curves \( y = \Gamma_1(x) \) and \( x = \Gamma_2(y) \) intersect at the unique communication-free equilibrium, \((\bar{x}, \bar{y})\). It is easy to check that \( \Gamma_2(y) < \Omega_2(y) \) whenever \( y \in (\xi, \Gamma_1(d_2)) \). Moreover,

\[ \tilde{\Omega}_2(\xi) = \Gamma_2(\xi) = \mu_2 \]

and

\[ \tilde{\Omega}_2(\Gamma_1(d_2)) = \Gamma_2(\Gamma_1(d_2)) < \Gamma_1(d_2) \]

where the inequality follows from the fact that \( \Gamma_2 \) is decreasing. These properties are drawn in Figure 6. Notice that the curve \( x = \tilde{\Omega}_2(y) \) lies to the right of the curve \( x = \Gamma_2(y) \) for all \( y \in (\xi, \Gamma_1(d_2)) \), but that the two curves intersect when \( y = \xi \) and \( y = \Gamma_1(d_2) \). Thus, a communication equilibrium does not exist. \( \square \)
Figure 1: Communication-free equilibrium with two coordination types

Figure 2: Communication-free equilibrium with a coordination type and an opportunistic type
Figure 3: Communication equilibrium with two coordination types

Figure 4: Communication equilibrium with two opportunistic types
Figure 5: No communication equilibrium with a coordination type and an opportunistic type (the hawkish extremist)

Figure 6: No communication equilibrium with a coordination type and an opportunistic type (the dovish extremist)
본 연구에서 Baliga and Sjöström(2012a)의 대칭적 의사결정자 간의 갈등게임을 비대칭적 의사결정자간의 게임으로 연장하면서, 북핵문제를 두고 발생하는 미국과 중국 간 갈등 상황에 적용한다. 본 연구의 갈등게임 모형은 대량살상무기를 가진 국가(극단주의적 성향의 국가)에 대해 ‘적극적 관여’ 또는 ‘비관여’의 상반되는 국제 외교 정책을 취하는 두 국가 간 국제 갈등 상황을 상정한다. 본 연구의 주요 결과는 다음과 같다. 첫째, 극단주의적 성향 국가의 대량살상무기에 의한 피해가 더 큰 국가가 ‘적극적 관여’를 선택할 확률이 높게 나타난다. 둘째, 두 국가의 관계가 전략적 보완 관계일 경우, 극단적 매파 성향의 국가가 공격적인 메시지를 보일 때 두 국가 모두 ‘적극적 관여’를 선택할 확률이 낮아진다. 셋째, 두 국가의 관계가 전략적 대체관계일 경우, 극단적 비둘기파 성향의 국가가 유화적 메시지를 보일 때 한 국가가 ‘적극적 관여’를 선택할 확률이 높아지는 반면, 다른 국가가 ‘적극적 관여’를 선택할 확률은 낮아진다. 마지막으로 본 연구는 위의 두 경우를 제외하고는 갈등게임에서 극단주의적 성향의 국가가 취할 수 있는 유효한 의사소통전략이 부재하다는 것을 증명한다.
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<table>
<thead>
<tr>
<th></th>
<th>Title</th>
<th>Authors</th>
</tr>
</thead>
<tbody>
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<tr>
<td>19-08</td>
<td></td>
<td></td>
</tr>
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Bargaining and War: On the Communication Equilibrium in Conflict Games

Youngseok Park and Colin Campbell

The players contemplate whether to take an active engagement action to compel the leader of a neighboring state (an extremist) to give up his risky weapons. We show that a player with greater damage from the extremist is more likely to choose an active engagement action than a player with lesser damage. The likelihood of both players choosing the active engagement action decreases by a hawkish extremist who can send a provocative message, if both players are coordination types. If both players are opportunistic types, a dovish extremist can send an appeasement message that causes one player to be more active while another to be more inactive.