The Rise of Software and Skill Demand Reversal*

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Abstract

The growth of high-skill jobs has stagnated since the 2000s, and this stagnation has been accompanied by a rise in software innovation relative to equipment. We provide empirical evidence that the occupational use of software or equipment is related to cognitive or routine tasks by occupation and propose a model that explains both employment share and software innovation trends. In the model, equipment-embodied technical change reduces the demand for routine tasks that intensively use equipment. Hence, the demand for equipment decreases, which leads to a rise in software innovation. This rise, in turn, generates a reversal in the demand for cognitive tasks.

Keywords: Job polarization, skill demand reversal, software innovation, endogenous sorting, directed technical change

JEL classification: E20, E22, J24

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1 Introduction

The occupational structure – the distribution of labor across occupations – has changed dramatically in the United States: the employment share of middle-skill jobs has declined since the 1980s (job polarization), and the growth of high-skill jobs has stagnated since the 2000s (skill demand reversal, Beaudry et al., 2016).

Many suggest recent technological change as a reason for the changes in the occupational structure: recent technological change has replaced routine tasks, and jobs in the middle are intensive in routine tasks. This paper suggests that the relation between technological change and occupational structure is bidirectional: technological change shapes the occupational structure, but changes in the occupational structure also affect the direction of the following technological change. Notably, the bi-directional relation can provide an explanation for both job polarization and skill demand reversal simultaneously.

We first document that skill demand reversal was accompanied by a higher growth in software innovation relative to equipment (figure 2.4a) and argue that this change in the direction of innovation was closely related to changes in the occupational structure of the U.S. economy. We do so by combining two datasets—the National Income and Product Account (NIPA) and O*NET Tools and Technology Database. The newly merged dataset shows that the average amount of investment in software and/or equipment by occupation are strongly linked to the tasks of each occupation. Namely, software is used intensively by cognitive (high-skill) occupations, while equipment is used intensively by routine (middle-skill) occupations (figure 2.6).

We then provide a unified framework that endogenously explains both skill demand reversal and the rise of software innovation. The model has three novel features. First, the model features workers of heterogeneous skill sorting into heterogeneous tasks, associated with different uses of two types of capital (software and equipment). Second, technological changes in the model are embodied into capital, and innovators endogenously choose which type of capital to innovate. Last but not least, all workers use two types of capital, but with different intensities, depending on their occupations. This situation departs from the typical assumption that only particular kinds of occupations are affected by a single type of technical change. These features enable us to simultaneously analyze the bi-directional interaction between the occupational structure and direction of technical change.

In the model, the intensities at which each occupation uses software and equipment can be measured directly from the newly merged dataset mentioned above. The production of goods occurs by combining occupational services, and they are provided by workers in each occupation using equipment and software. Equipment and software are modeled as a composite of infinitesimal varieties provided by innovators, who are free to choose a type of capital to innovate.

After characterizing the equilibrium, we prove a series of analytical comparative statics in response to one exogenous change: an increase in the productivity of the equipment-producing sector.[1] Increased productivity in equipment production makes middle-skill occupations more productive than others since middle-skill occupations use equipment most intensively. As a result, labor flows out from these jobs and into high- and low-skill jobs when occupational services are complementary in production. However, the decline in middle-skill employment also means that the demand for equipment declines, inducing innovators to shift their focus away from equipment and more toward enhancing software. In turn, the rise of software leads to skill demand reversal if jobs are complementary: middle-skill jobs were already declining (job polarization), the employment share of high-skill jobs decelerates since these jobs use software most intensively, and consequently, skill demand becomes concentrated in low-skill jobs.

The complementarity between occupational services implies that the demand for certain occupations can be lowered when improving a type of capital used heavily by the occupations. Importantly, software capital is heavily used by cognitive (not routine) task-intensive occupations. For example, an introduction of 3D modeling software helps architects work more quickly. When the demand for 3D modeling tasks does not increase enough to compensate for the increase in the productivity of architects, the demand for architects should decrease.

[1]We also document a faster increase in the productivity of the equipment-producing sector in the data.

One may wonder how we can distinguish software from equipment as software is generally embodied in equipment. Equipment is necessary for the use of software, but it can install a variety of software, and the variety itself can change over time. An operational software usually comes with the purchase of a PC, but many additional software can be installed for different types of tasks afterwards. The above example of 3D modeling software also illustrates the software innovation we highlight in this paper, distinguished from equipment innovation. In fact, the software share of intermediate input in equipment production has been roughly constant over three decades, while the software investment relative to equipment investment has increased dramatically (figure 2.5).

We verify the empirical validity of the model's mechanism using the fact that the decline in the relative price of equipment to software varies across industries. The model predicts: i) a negative relationship between the speed of decline in the relative price of equipment to software and the growth of middle-skill employment relative to high-skill employment and ii) a positive correlation between the speed of decline in the relative price of equipment to software and the relative growth of software innovation to R&D other than software. We confirm significant correlations in both cases.

Confident of the mechanism, we use the model to quantify its importance. Our quantitative analysis shows that the channel of directed technical change can account for 70 to 80% of the rise in software and skill demand reversal, where the latter is measured by the gap between the actual series and the level implied by the linear trend of the 1980s.

The results have two important implications. First, technological change and the occupational structure have a bi-directional interaction. Technical progress shapes occupational demand, but this change in the occupational demand can also lead to another type of technical change that generates a different occupational structure. Though the importance of task-specific technological change (or skill-biased technological change) has been emphasized in the literature, the source of those technologies has been investigated to a lesser extent. This paper addresses this by analyzing two different types of innovation endogenously responding to changes in the occupational structure.

Second, software and equipment capital measured in the National Accounts is a good proxy for the technological changes that shape the structure of the labor market. By linking recent technological changes to different types of capital, we bring the directional technical change mechanism to an empirical dimension. Since technological changes have significant impacts on many economic variables, careful investigation of the composition of capital investment can be fruitful for understanding the measured impact of recent technological changes.

Related Literature The relationship between polarization and increases in the productivity of middle-skill occupations, which are intensive with respect to routine tasks, is well documented in the literature (e.g., Autor et al., 2006; Autor and Dorn, 2013; Goos et al., 2014, among others). Fewer studies have discussed the puzzling observation that there has been a flattening of the demand for high-skill workers around 2000, known as "Great Reversal (or skill demand reversal)" (Beaudry et al., 2016; Valletta, 2016).

In particular, Beaudry et al. (2016) was the first to document the reversal and suggests an explanation based on a technology boom-bust cycle. When the implementation of new technology has the form of capital investment, an introduction of new technology initially has a high level of marginal productivity and therefore generates a boom in the demand for cognitive occupations. As the marginal productivity of the technology decreases, the demand for highly paid jobs also starts to decrease.

Our paper suggests that it could be a change in the direction of technology generating the skill demand reversal, not a stopping of investment. Actually, the relative importance of software development relative to other types of R&D has continually increased (figure 2.4). In addition, we see a continuation of decreasing middle-paying jobs in the 2000s with a much greater increase in the low-paying jobs (figure 2.3). If the reversal happened because we entered the maturity stage of new technology around 2000, we might have seen a more stablized occupational structure than observed in the data.

Several papers analyze the consequences of task-specific technological change on the labor market with an assignment model (Costinot and Vogel, 2010; Lee and

Shin, 2017; Michelacci and Pijoan-Mas, 2016; Stokey, 2016; Cheng, 2017, among others). We have a similar assignment feature but characterize tasks by their different uses of two types of capital, and we also introduce endogenous task-specific technological change generated from innovations on each type of capital. By doing so, we obtain a direct mapping of two distinctive task-specific technological changes to observed data.

Krusell et al. (2000) also links the price of equipment capital to skill-biased technical change and emphasizes that skill-capital complementarity (capital substitutes low-skill labor more than high-skill labor) is key to understanding how a rise in the productivity of capital leads to higher demand for high-skill workers. In our model, the substitutability between labor and capital is the same across occupations. We assume that occupations vary in how intensively they use different types of capital and that the occupations are complementary. Still, our model nests the model of Krusell et al. (2000) as a special case – a version of two types of tasks, with one task having zero intensity of capital.

While Krusell et al. (2000) classifies workers by education, we classify workers by occupation. High-educated workers may well be able to do what less educated workers usually do, whereas tasks performed by certain occupations may not be able to be substituted by tasks by other occupations. Indeed, recent papers such as Goos et al. (2014) and Lee and Shin (2017) highlight complementarity between tasks as key to understanding task-level employment changes (i.e., polarization). In this regard, our paper complements Krusell et al. (2000) by linking capital-embodied technical change to occupational employment.

Cheng (2017) measures the routine-biased technological change from the different capital intensities across occupations. Unlike ours, Cheng (2017) measures capital intensities across occupations from the industry-level capital share and variations in the composition of occupations across industries and confirms that middle-skill occupations are capital intensive. We show, however, that the distinction between equipment and software is important, as software is not used intensively in middle-skill jobs. In addition, we explain why a particular type of technology may or may not respond.

One of the main features in this paper is that the relation between technological change and labor market outcome is bi-directional, and therefore the evolution of technology is partially endogenous. Acemoglu and Restrepo (2016) and Hémous and Olsen (2016) also analyze the interaction between technological change and the labor market with the directed technical change framework of Acemoglu (2002). These authors provide new insights on how automated technology evolves and affects labor market outcomes, but the interpretation of technology with respect to the observable data is not straightforward, making its empirical test diffcult. Our technological changes are directly measured from investment in software and equipment in the National Accounts, and tasks are mapped to occupations, facilitating the quantification of the model.

Recent studies by Bárány and Siegel (forthcoming) and Lee and Shin (2017) show that either task-specific technological change or sector-specific technological change can lead to both job polarization and structural change. Since a single type of technological change can result in both phenomena, it is not easy to conclude whether the source of technological change is task or sector specific. Our paper implies that the technological change embodied in a particular type of capital could be a source of task-specific technological change that can generate both phenomena simultaneously.

Another important feature of this paper is distinguishing software capital from equipment capital. Software investment is becoming increasingly important, as evidenced by its rapid rise as a share of total investment. Aum et al. (2018) analyzes the role of computer capital (hardware and software) in shaping the dynamics of aggregate productivity. Koh et al. (2016) emphasizes the importance of software capital (more broadly, intellectual property product capital) in accounting for the declining labor share in the US. Our model also generates a decline in the labor share as a result of higher software investment, and we show that there is a significant correlation between a decline in the labor share and software intensity at the industry level.

The rest of the paper is organized as follows. In section 2, we summarize the relevant empirical facts. In section 3, we present the model and characterize its equilibrium. In section 4, we conduct analytical comparative statics and in section 5, we verify that the model's predictions hold empirically across industries. In section 6, we calibrate the model to quantify how important its mechanism is for accounting for the rise of software and skill demand reversal. Section 7 concludes.

2 Key Facts

We document several data observations. First, equipment-producing industries have experienced much faster total factor productivity (TFP) growth than software-producing industries. Second, the secular pattern of polarization shows that the rise of high-skill occupations slowed with a greater increase in low-skill occupations since the mid-1990s. Third, software development expenditures have increased relative to equipment-related R&D expenditures, especially since the mid-1990s. Fourth, the intensity of equipment and software across tasks is closely correlated with routine task intensity and cognitive task intensity.

2.1 Sectoral Productivities: Equipment vs Software

The input-output table published by BEA reports the industrial composition of each type of capital investment annually. From the table, we obtain the weights on detailed industries that contribute to the production of equipment and software investment goods for every year[2]. The productivities of equipment- and software-producing industries are then computed as a weighted average of individual-level industrial TFP, on the basis of their contribution. Specifically, we compute the total factor productivity (TFP) of equipment- and software-producing industries according to the Törnqvist index.

Using BEA's Industry Account, an industry i's TFP growth between time u and t can be computed as

$$\log(TFP_{i,t}/TFP_{i,u}) = \log(y_{i,t}/y_{i,u}) - \frac{\alpha_{i,t} + \alpha_{i,u}}{2} \log(k_{i,t}/k_{i,u}),$$

[2]For example, equipment investment goods are a composite of 22 commodities out of 61 commodities, classified under the BEA industry code.

where y is the real value added per employment, k is the real non-residential capital divided by the number of employment, and α is one minus the labor share.

From the input-output table of each year (t), we observe the composition of commodities (industries) that consists of equipment and software, respectively. Let I be the total number of industries. Then we can represent equipment and software as

$$p_t E_t = \sum_i^I p_{i,t} E_{i,t}, \quad p_t S_t = \sum_i^I p_{i,t} S_{i,t},$$

where E_i or S_i is 0 when industry i's output is not a part of equipment or software. Then the share of industrial output in equipment or software is obtained as

$$\omega^e_{i,t} \equiv p_{i,t} E_{i,t} / p_t E_t, \quad \omega^s_{i,t} \equiv p_{i,t} S_{i,t} / p_t S_t.$$

The TFPs of the equipment- and software-producing industries are computed by

$$\log(TFP_{e,t}/TFP_{e,t-1}) = \sum_i \frac{\omega^e_{i,t} + \omega^e_{i,t-1}}{2} \log(TFP_{i,t}/TFP_{i,t-1}),$$
$$\log(TFP_{s,t}/TFP_{s,t-1}) = \sum_i \frac{\omega^s_{i,t} + \omega^s_{i,t-1}}{2} \log(TFP_{i,t}/TFP_{i,t-1}).$$

The results are presented in figure 2.1. The figure shows that equipment has been made much more productively than software since 1980.

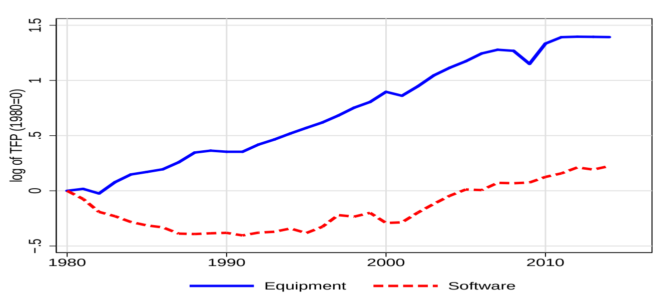


Fig. 2.1: TFP of equipment- / software-producing industries

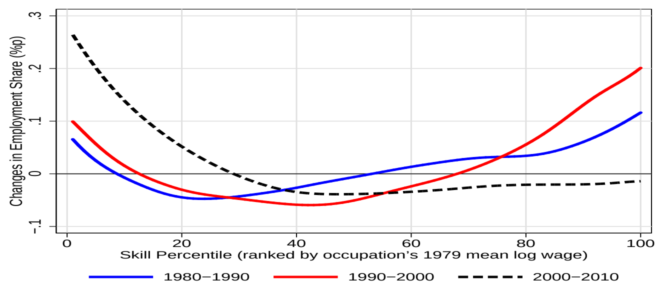


Fig. 2.2: Changes in employment structure in the US by decade

Note: Each point on the horizontal axis is a group of occupations composing 1% of total employment in 1980, sorted by the 1979 average log wage.

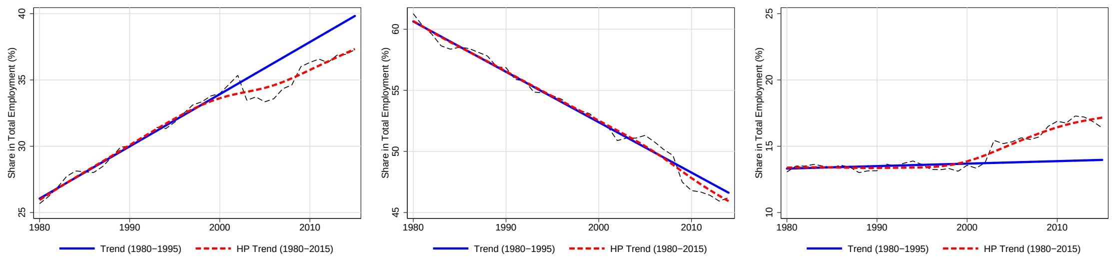
2.2 The Pattern of Job Polarization

Figure 2.2 shows changes in the employment share across skill percentile by decade from 1980, computed from Census/ACS data. Each point in the skill percentile represents a group of occupations representing 1% of the labor supply in 1980, sorted by the average log hourly wage in 1979.

The figure shows clear U-shaped changes in employment share from 1980 to 2010. By assessing the three lines separately, however, we see that the rise in high-skill occupations is strongest in the first two decades, while that of low-skill occupations accelerates during 2000-2010. Moreover, the range of shrinking occupations shifts toward the right across decades.

Similar observations are also in the annual data from CPS when occupations are classified into three groups: cognitive (high-skill), routine (middle-skill), and manual (low-skill)[3]. We compare two different trends – a linear trend from 1980 to 1995 and an HP trend from all data points – of the employment share of each occupational group. Figure 2.3 confirms that there were breaks in the trends of employment shares of cognitive occupations and manual occupations in the mid-1990s. Notably, the decline in routine occupations continued until recently.

[3] The classification of occupations is based on the one-digit SOC. Cognitive occupations are management, professionals, and technicians. Routine occupations are machine operators, transportation, sales and office, mechanics, and miners and production.



(a) Cognitive (high-skill) (b) Routine (middle-skill) (c) Manual (low-skill)

Fig. 2.3: Employment share of cognitive, routine, and manual occupations
Note: 1) Cognitive occupations are management, professionals, and technicians. Routine occupations include office and sales, transportation, machine operators, mechanics, construction and production workers.
2) The blue line is the linear trend from 1980 to 1995, and the red (dashed) line is the HP trend with a smoothing parameter of 100. All vertical axes represent 15% of the range.

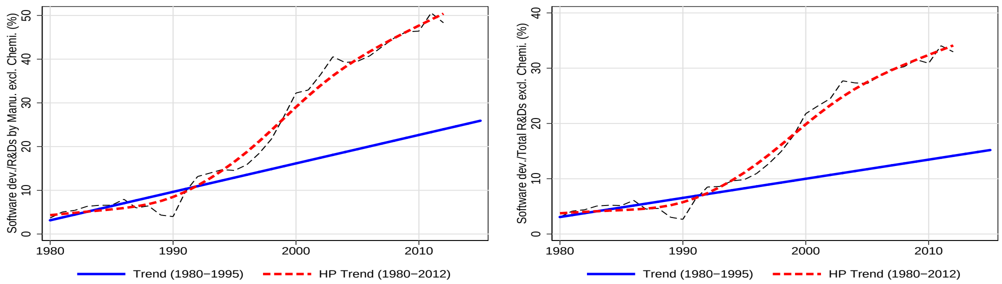
2.3 Rising Software Innovation

We now turn to the R&D composition in the US. Software development expenditures can be obtained from Crawford et al. (2014) or from differences between R&D in NIPA, excluding software development and R&D recorded in the innovation satellite account, which includes software development.

Figure 2.4a shows the size of software development relative to R&D expenditures funded by the manufacturing sector, excluding chemical-related R&D, [4] across years. As a robustness check, we also assess the relative size of software development relative to all other R&Ds only excluding chemicals (figure 2.4b). Both show an increasing pattern, especially since the mid-1990s, suggesting that changes in the pattern of polarization could be related to increasing software innovation.

Later, we argue that the increasing innovational expenditures in software exhibit a change in the direction of technical progress away from equipment to software. One may think that software and equipment are difficult to distinguish as early innovation in equipment is successful only after combined with software. While innovative equipment also uses software, operational software has a differ-

[4]We view R&D expenditures funded by the manufacturing sector excluding chemical-related industries as the expenditures most closely related to R&D on equipment.



(a) Relative to R&D's by manufact.[1)] (b) Relative to total R&D's[1)]

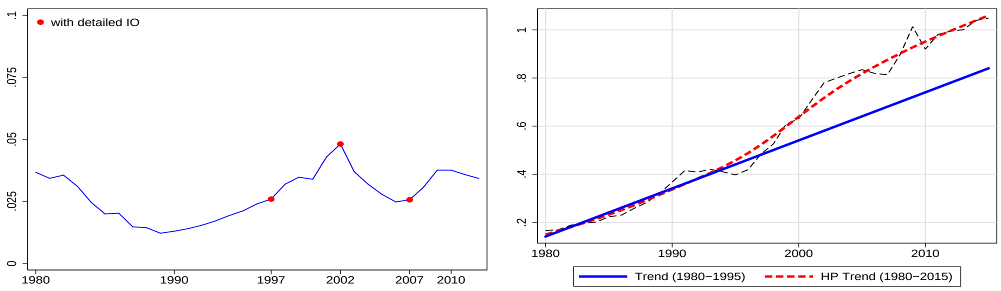
Fig. 2.4: Software innovation

Note: 1) We exclude chemical industries as it is least related to capital innovation. 2) The blue line is the linear trend from 1980 to 1995, and the red (dashed) line is the HP trend with a smoothing parameter of 100.

ent role from the software used directly by workers for certain tasks. Operational software only helps in the task that can be directly done through (many different types of) machines.

Operational software is usually installed in a stage of machine production; hence, it can be captured as an intermediate input of the equipment-producing industries. In contrast, the software used directly by workers – such as accounting software, designing software, enterprise resource planning, and statistics packages – is usually installed after the production of a machine (usually PC), and should be captured as investment by firms. Indeed, the amount of software used as an intermediate input in the production of equipment[5] has been roughly constant over time, while the software investment relative to IT equipment has increased, especially since mid 1990s (figure 2.5), suggesting that the increase in software investment captures software development supporing workers' tasks directly, distinguished from an innovation enabling fast operation of a machine.

[5] We obtain intermediate consumption of software commodities, 511200 (software publishers) and 541511 (custom computer programming services) from a detailed Input-Output table, available for the year 1997, 2002, and 2007. Then, we linearly inter- and extra-polate the ratio of commodities 511200 to 511 and 541511 to 5415 to periods of 1980 to 2014. By multiplying the estimated ratio to intermediate use of industries 511 and 5415, we estimate software intermediates for 1980 and 2014.



(a) Software intermediate in equipment (b) Software investment to equipment investment[1)]

Fig. 2.5: Software: Used for the operation of equipment (intermediate) versus used for workers' tasks (investment)

Note: 1) The blue line is the linear trend from 1980 to 1995, and the red (dashed) line is the HP trend with a smoothing parameter of 100.

2.4 Capital Use by Occupation

We provide data evidence to document strong connections between the use of different types of capital across occupations. Specifically, we construct capital use by occupation by combining two data sources – NIPA and the O*NET Tools and Technology Database.

The O*NET Tools and Technology database provides information about the types of tools and technology (software) used by each occupation. One caveat of this dataset is that it does not provide information about the price of each item. To address this shortcoming, we attempt to link capital items in O*NET Tools and Technology to the NIPA data obtained from the Bureau of Economic Analysis (BEA).

Specifically, we make a naive concordance between the UN Standard Product and Services Code (UNSPSC), a product classification system used in the O*NET database, and 25 categories of non-residential equipment in NIPA table 5.5 (details can be found in the appendix A). Then, we distribute the total amount of a particular type of equipment investment to each occupation by means of the number of tools included in the investment category, according to the concordance. By doing this, we assign value information to the number of tools used by the occupation.[6]

For example, suppose that firms have invested USD 20 billion in metalworking machinery in the NIPA table. According to the constructed concordance, metalworking machinery includes a total of 139 commodities in the UNSPSC. Some occupations use none of the 139 commodities, while other occupations use various amounts of the commodities in the category. Because we know the amount of employment by occupation, we can calculate the total number of metalworking machinery items used by all workers in a given year. Then, we can approximate the amount attributed to an individual occupation by distributing the total USD 20 billion investment according to the number of items used by the occupation. Subsequently, dividing by the number of employees provides an estimate of the per capita investment in metalworking machinery by occupation.

The per capita investment in equipment by occupational skill group is shown in figure 2.6a, where an occupational skill group is defined as a group representing 1% of total employment among all occupations ranked by mean hourly wages. We also plot the routine-intensive task share – the share of routine-intensive employment out of total employment within the skill group – in the same figure. Here, routine-intensive employment is defined as employment in occupations with the highest one-third routine task index of all occupations, where the routine task index is computed using the O*NET task database following Acemoglu and Autor (2011).

In figure 2.6b, we plot software investment per capital across the same wage percentile and the cognitive-intensive task share defined similarly to the routine-intensive task share. Again, the cognitive task index is computed following Acemoglu and Autor (2011).

We can see from the figures that middle-skill workers use equipment more intensively, whereas high-skill workers use software more intensively. Moreover, the use of equipment closely follows the routine task share, while the use of software is closely related to the cognitive task share. We also illustrate the use of

[6]Of course, this does not provide perfect information on the value of tools as it only differentiates an average price across BEA categories. As long as average prices in each category of BEA classification have a meaningful dispersion, however, this method can alleviate bias from ignoring all the price information.

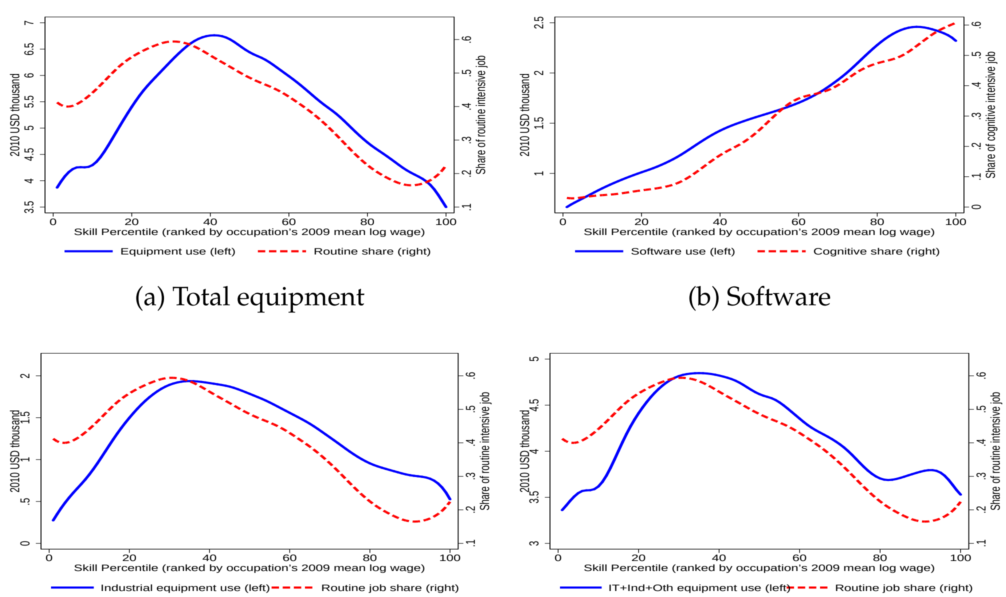
the equipment subitem by occupation in figures 2.6c (industrial equipment) and 2.6d (industrial and information processing equipment). Among the equipment subitems, industrial equipment is most strongly correlated with the routine task intensity.

3 Model

Now, we present a model capturing the facts in the previous section. In particular, we aim to capture endogenous interactions between the occupational structure and the directional technical change.

3.1 Environment

There is a continuum of individuals endowed with human capital $h \in [1, \bar{h}]$ drawn from a measure $\mathcal{M}(h)$. Specifically, we assume that



(a) Total equipment

(b) Software

(c) Industrial equipment

(d) Industrial + IT + Other equipment

Fig. 2.6: Use of equipment and software across skill percentile

Note: Detailed information on the data is provided in appendix A.

Assumption 1 (distribution) *The measure of skill,* $\mathcal{M} : [1, \bar{h}] \mapsto [0, 1]$, *is a cumulative distribution function with a differentiable probability distribution function,* $\mu : [1, \bar{h}] \mapsto \mathbb{R}+$.

There is a continuum of tasks $\tau \in [0, \bar{\tau}]$, and final goods are produced by combining task output $T(\tau)$ according to

$$Y = \left(\int_{\tau} \gamma(\tau)^{\frac{1}{\epsilon}} T(\tau)^{\frac{\epsilon-1}{\epsilon}} d\tau \right)^{\frac{\epsilon}{\epsilon-1}}. \quad (1)$$

The task output is produced by integrating human-capital-specific task production $y(h, \tau)$ across all skill levels used for the production of task τ:

$$T(\tau) = \int_{h \in \mathcal{L}(\ll)} y(h, \tau) dh. \quad (2)$$

The human-capital-specific task production, $y(h, \tau)$, depends not only on worker human capital h but also on task τ that the worker is performing. Specifically, the functional form of $y(h, \tau)$ is given by

$$y(h, \tau) = \left[\left\{ \alpha_h(\tau) \left(b(h, \tau) l \right)^{\frac{\sigma_e - 1}{\sigma_e}} + \alpha_e(\tau) E^{\frac{\sigma_e - 1}{\sigma_e}} \right\}^{\frac{\sigma_e(\sigma_S - 1)}{(\sigma_e - 1)\sigma_S}} + \alpha_S(\tau) S^{\frac{\sigma_S - 1}{\sigma_S}} \right]^{\frac{\sigma_S}{\sigma_S - 1}}, \quad (3)$$

where $l(h)$ represents the level of employment of workers with human capital h and S and E represent software and equipment, respectively.

The function $b(h, \tau)$ captures the productivity of a worker with human capital h when she performs a task τ. We assume that $b(h, \tau)$ is strictly log supermodular.

Assumption 2 *The function* $b(h, \tau) : [1, \bar{h}] \times [0, \bar{\tau}] \mapsto \mathbb{R}^+$ *is differentiable and strictly log supermodular. That is,*

$$\log b(h', \tau') + \log b(h, \tau) > \log b(h, \tau') + \log b(h', \tau),$$

for all $h' > h$ *and* $\tau' > \tau$.

As shown in Costinot and Vogel (2010), assumption 2 helps to ensure positive assortative matching (PAM). In other words, as human capital h increases, the higher τ task she will perform in equilibrium. Not only how each occupation utilizes a worker's human skill but also tasks are different with regard to the intensities in which they use two types of capital. This second feature is essential to understanding the differential effects of capital-embodied technical change on various occupations. [7]

The software and equipment available for workers are given by

$$S = \left(\int_0^{N_s} s(k)^{\nu_s} dk \right)^{\frac{1}{\nu_s}} \text{ and } E = \left(\int_0^{N_e} e(k)^{\nu_e} dk \right)^{\frac{1}{\nu_e}}, \tag{4}$$

where each variety of capital ($s(k)$ and $e(k)$) is provided by a permanent patent owner under monopolistic competition.

The production technology of software or equipment is

$$s(k) = A_s x, \quad e(k) = A_e x,$$

where x is the amount of final goods used to produce software or equipment. The production technology implies that the marginal costs of producing software and equipment are given by the inverse of productivity, $q_s := 1/A_s$ and $q_e := 1/A_e$.

New software and equipment are created from R&D expenditures Z_s and Z_e, and the laws of motion for total varieties follow

$$\dot{N}_s = Z_s/\eta_s \text{ and } \dot{N}_e = Z_e/\eta_e. \tag{5}$$

Finally, the representative household has a CRRA preference given by

$$\int_s^{\infty} e^{-\rho t} \frac{C(t)^{1-\theta} - 1}{1-\theta} dt,$$

[7] The production structure allows differential effects of capital-embodied technical change on various occupations for three reasons. First, each occupation utilizes human skill differently. Second, occupations rely on capital with various intensities. Third, any changes in the relative productivity at the occupation level alter relative demand for occupations through the final production, combining all tasks.

and the resource constraint in the economy is

$$C + q_e \int_0^{N_s} s(k)dk + q_s \int_0^{N_e} e(k)dk + Z_e + Z_s \leq Y. \tag{6}$$

3.2 Static Equilibrium

To characterize the static equilibrium, we take the total varieties of software and equipment, N_e and N_s, as given. We first define the equilibrium.

Definition 1 (Static equilibrium) *The static equilibrium consists of the price function $p(\tau)$, $w(h)$, $p_s(k)$, and $p_e(k)$, the quantity function $T(\tau)$, $l(h,\tau)$, $s(k,\tau)$, $e(k,\tau)$, and the quantity Y such that*

1. *Given $p(\tau)$, the final goods producer solves*

$$\max \; Y - \int_\tau p(\tau)T(\tau)d\tau,$$

   given equation (1).

2. *For each task, the task output is produced to solve*

$$\max \; p(\tau)T(\tau) - \int_h w(h)l(h,\tau)dh - \int_0^{N_s} p_s(k)s(k,\tau)dk - \int_0^{N_e} p_e(k)e(k,\tau)dk,$$

   given equation (2), $w(h)$, $p_s(k)$, and $p_e(k)$.

3. *A capital provider solves*

$$\max \; \pi_s(k) = \int_\tau \left[p_s(k)s(k,\tau) - q_s s(k,\tau)\right] d\tau,$$
$$\max \; \pi_e(k) = \int_\tau \left[p_e(k)e(k,\tau) - q_e e(k,\tau)\right] d\tau,$$

   given the marginal cost q_s and q_e.

4. *All workers choose the highest-paying occupation (task).*

5. *The labor market clears $\mu(h) = \int_\tau l(h,\tau)d\tau$.*

From the final goods production, the demand for task output $T(\tau)$ is given by

$$p(\tau) = \left(\frac{\gamma(\tau)Y}{T(\tau)}\right)^{\frac{1}{\epsilon}}, \tag{7}$$

and the price function $p(\tau)$ satisfies $\int_\tau \gamma(\tau)p(\tau)^{1-\epsilon}d\tau = 1$.

Since we assume that the capital producer maximizes profit under monopolistic competition, we obtain the price of the software and equipment as

$$p_s(k) = \frac{1}{A_s\nu_s} \text{ and } p_e(k) = \frac{1}{A_e\nu_e}, \text{ for all } k.$$

By substituting this result into the first-order conditions from task output production, we can show that the wage function $w(h)$ satisfies

$$w(h) \geq \underbrace{\left[\left\{p(\tau)^{1-\sigma_s} - \left(\frac{\alpha_s(\tau)^{\frac{\sigma_s}{1-\sigma_s}}}{A_s N_s^{\varphi_s}\nu_s}\right)^{1-\sigma_s}\right\}^{\frac{1-\sigma_e}{1-\sigma_s}} - \left(\frac{\alpha_e(\tau)^{\frac{\sigma_e}{1-\sigma_e}}}{A_e N_e^{\varphi_e}\nu_e}\right)^{1-\sigma_e}\right]^{\frac{1}{1-\sigma_e}} \alpha_h(\tau)^{-\frac{\sigma_e}{1-\sigma_e}}}_{:=\omega(\tau)}$$
$$\times b(h,\tau), \tag{8}$$

with equality when $l(h,\tau) > 0$.

Equation (8) shows that the wage function $w(h)$ can be expressed as a product of terms depending only on τ ($\omega(\tau)$) and human capital task-specific productivity $b(h,\tau)$. The existence of PAM between h and τ follows.

Lemma 1 (Positive assortative matching) *Under assumptions 1 and 2, there exists a continuous and strictly increasing assignment function $\hat{h} : [0,\bar{\tau}] \mapsto [1,\bar{h}]$ such that $\hat{h}(0) = 1$ and $\hat{h}(\bar{\tau}) = \bar{h}$.*

The proof is same as the proof of Lemma 1 in Costinot and Vogel (2010) and is omitted.

The equilibrium assignment $\hat{h}$ is characterized by

Lemma 2 (Equilibrium assignment function) *The equilibrium assignment function $\hat{h}(\tau)$, price function $p(\tau)$, and the wage rate $\omega(\tau)$ satisfy the following system of differ-*

ential equations.

$$\frac{d\log\omega(\tau)}{d\tau} = -\frac{\partial \log b(\hat{h}(\tau),\tau)}{\partial\tau}, \tag{9}$$

$$\hat{h}'(\tau) = \frac{\gamma(\tau)p(\tau)^{\sigma_s-\epsilon}\alpha_h(\tau)^{\sigma_s}\psi(\tau)^{\sigma_e-\sigma_s}Y}{\omega(\tau)^{\sigma_e}b(\hat{h}(\tau),\tau)\mu(\hat{h}(\tau))}, \tag{10}$$

$$p(\tau) = \left[\psi(\tau)^{1-\sigma_s} + \alpha_s(\tau)^{\sigma_s}\left(\nu_s A_s N_s^{\varphi_s}\right)^{\sigma_s-1}\right]^{\frac{1}{1-\sigma_s}}, \tag{11}$$

with $\hat{h}(0)=1$, $\hat{h}(\bar{\tau})=\bar{h}$, $\int\gamma(\tau)p(\tau)^{1-\epsilon}d\tau = 1$,
$\psi(\tau) := \left[\alpha_h(\tau)^{\sigma_e}\omega(\tau)^{1-\sigma_e} + \alpha_e(\tau)^{\sigma_e}\left(\nu_e A_e N_e^{\varphi_e}\right)^{\sigma_e-1}\right]^{\frac{1}{1-\sigma_e}}$, $\varphi_e := \frac{1-\nu_e}{\nu_e}$, *and* $\varphi_s := \frac{1-\nu_s}{\nu_s}$.

Proof *In appendix C.* ∎

After the assignment function $\hat{h}$ is obtained, all the equilibrium quantities and prices can be computed.

3.3 Dynamic Equilibrium

Now consider a dynamic equilibrium where technology evolves endogenously. The HJB equations for innovators are given by

$$r(t)V_s(k,t) - \dot{V}_s(k,t) = \pi_s(k,t), \tag{12}$$

$$r(t)V_e(k,t) - \dot{V}_e(k,t) = \pi_e(k,t), \tag{13}$$

with profit functions,

$$\pi_s(k) = \int_\tau [p_s(k)s(k,\tau) - q_s s(k,\tau)]d\tau = \frac{1-\nu_s}{\nu_s A_s}\int_\tau s(k,\tau)d\tau, \tag{14}$$

$$\pi_e(k) = \int_\tau [p_e(k)e(k,\tau) - q_e e(k,\tau)]d\tau = \frac{1-\nu_e}{\nu_e A_e}\int_\tau e(k,\tau)d\tau. \tag{15}$$

The free entry condition ensures that

$$V_e \le \eta_e, \text{ with equality if } Z_e > 0, \quad \text{and} \quad V_s \le \eta_s, \text{ with equality if } Z_s > 0.$$

If both R&D's are positive, we have $\eta_e V_e = \eta_s V_s$, and from equations (12) and (13),

$$r(t) = \pi_e(t)/\eta_e = \pi_s(t)/\eta_s. \tag{16}$$

Finally, from the household's problem, we have a standard Euler equation:

$$\frac{\dot{C}(t)}{C(t)} = \frac{r(t) - \rho}{\theta}, \tag{17}$$

and the transversality condition:

$$\lim_{t \to \infty} \left[e^{-\int_0^t r(s)ds} \left(N_e(t)V_e(t) + N_s(t)V_s(t) \right) \right] = 0.$$

Now, we have a characterization of the steady state equilibrium in the following lemma.

Lemma 3 (Steady state equilibrium) *There exist sufficiently large $\nu_e < 1$ and $\nu_s < 1$ that are compatible with the unique steady state equilibrium, i.e.,*

$$\pi_e/\eta_e = \pi_s/\eta_s = \rho, \tag{18}$$

and every variable remains constant. Moreover, when $\sigma_s = \sigma_e = 1$, $\max\left\{ \frac{1-\nu_s}{\nu_s}\frac{\alpha_s(\tau)}{\alpha_h(\tau)} + \frac{1-\nu_e}{\nu_e}\frac{\alpha_e(\tau)}{\alpha_h(\tau)} \right\} < 1$ ensures the existence of the steady state equilibrium.

Proof *In appendix C.* ∎

Intuitively, high enough ν_e and ν_s ensure profits by providing additional variety that is not too great, making the rate of return on increasing variety strictly decreasing on the total varieties. As the rate of return is strictly decreasing in the size of varieties, we have a certain level of variety that equates the rate of return and time preference (ρ), leading to the existence of the steady state.

We consider only a case with a no-growth steady state because no standard balanced growth path exists when the task production is a general CES function. Note that the source of growth (increasing variety) is a capital-augmented technological change in our model. It is well known that no balanced growth path would exist for a capital-augmented technical change if the production function is not of the Cobb-Douglas form (e.g., Grossman et al., 2017). [8]

Exogenous vs Endogenous Productivity Our model has both exogenous and endogenous productivity for equipment and software. Exogenous productivity is augmented in capital production, A_e or A_s, and captures how well one can produce equipment or software that has already been introduced. For example, as equipment production becomes faster as a result of using a faster computers in the machine production process, this phenomenon would be captured in the increase in A_e. Instead, endogenous productivity, N_e or N_s, captures an introduction of new types of capital into the economy. For example, the development of the 3D modeling application supporting architects would be captured by an increase in N_s.

4 Comparative Statics

In this section, we restrict our attention to the case with $\sigma_e = \sigma_s = 1$, $\eta_e = \eta_s$ and $\nu_e = \nu_s$ to obtain analytical comparative statics. Specifically, we assume

Assumption 3 *The elasticities of substitution between labor and capital are 1, i.e., $\sigma_s = \sigma_e = 1$. The individual task production function is then*

$$y(h,\tau) = (b(h,\tau)l(h))^{\alpha_h(\tau)} E^{\alpha_e(\tau)} S^{\alpha_s(\tau)}.$$

Additionally, we put several structures on the intensity functions $\alpha_h(\tau)$, $\alpha_e(\tau)$, and $\alpha_s(\tau)$ to reflect the fact that high-skill workers use software intensively and middle-skill workers use equipment intensively, i.e.,

Assumption 4 (intensities) *The functions $\alpha_h(\tau) : [0,\bar{\tau}] \mapsto (0,1]$, $\alpha_s(\tau) : [0,\bar{\tau}] \mapsto (0,1]$ and $\alpha_e(\tau) : [0,\bar{\tau}] \mapsto (0,1]$ satisfy the following.*

[8] In the Cobb-Douglas task production case ($\sigma_s = \sigma_e = 1$); however, sustained growth can be obtained by assuming strictly positive population growth, as in Jones (1995). Every task still grows at a different rate, so the most labor-intensive task (the slowest-growing task) would dominate the economy in the limit under complementarity between tasks ($\epsilon < 1$), which is similar to the results in Ngai and Pissarides (2007) and Acemoglu and Guerrieri (2008).

2.1 $\alpha_s(\tau)$ *is differentiable and increasing on* $[0, \bar{\tau}]$.

2.2 $\alpha_e(\tau)$ *is differentiable, increasing on* $[0, \tau_e]$ *and decreasing on* $[\tau_e, \bar{\tau}]$.

2.3 $\alpha_e(\tau_e) > \alpha_s(\tau_e)$, $\alpha_s(\bar{\tau}) > \alpha_e(\bar{\tau})$, *and* $\alpha_e(0) = \alpha_s(0)$.

Now, we show that an increase in the productivity of equipment production ($A_e \uparrow$) leads to polarization and the rise of software and skill demand reversal when the tasks are complementary. Specifically, we focus on three main predictions of the model: (1) the polarization induced by the rise of equipment-producing productivity in the static equilibrium, (2) the subsequent rise of software innovation, and (3) the decreasing demand for high-skill employment in the steady state.

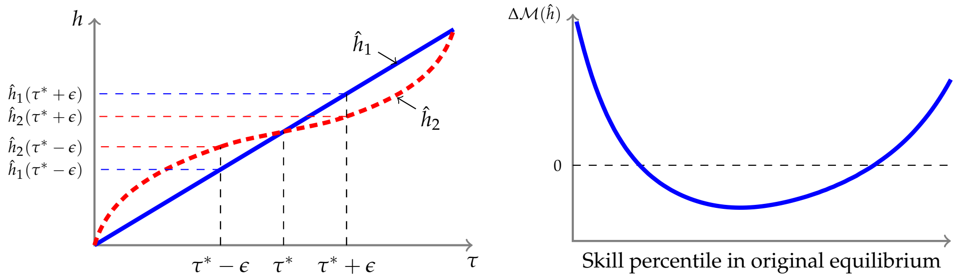
Job Polarization First, we show the impact of an increase in the equipment productivity (A_e) on the equilibrium assignment function $\hat{h}(\tau)$ in the static equilibrium (i.e., when N_e and N_s are fixed). We consider $A_{1e} < A_{2e}$ and denote the equilibrium assignment functions corresponding to A_{1e} and A_{2e} as $\hat{h}_1$ and $\hat{h}_2$, respectively.

Proposition 1 (Polarization) *Consider* $A_{1e} < A_{2e}$. *Suppose* $\epsilon < 1$ *and assumptions 1 to 4. For sufficiently small* $\alpha_h'(\tau)$, *we have* $\tau^* \in (0, \bar{\tau})$ *such that* $\hat{h}_1(\tau^*) = \hat{h}_2(\tau^*)$, $\hat{h}_1(\tau) < \hat{h}_2(\tau)$ *for* $\tau \in (0, \tau^*)$, *and* $\hat{h}_1(\tau) > \hat{h}_2(\tau)$ *for* $\tau \in (\tau^*, \bar{\tau})$.

Proof *In appendix C.* ∎

Proposition 1 states that there will be a shrinking task employment around τ^* where corresponding equipment intensity $\alpha_e(\tau^*)$ is relatively higher than $\alpha_e(0)$ and $\alpha_e(\bar{\tau})$. Figure 4.1 illustrates the change in the assignment function with A_{1e} (blue solid line) and $A_{2e} > A_{1e}$ (red dashed line). For a given task $\tau \in [\tau^* - \epsilon, \tau^* + \epsilon]$, we can see that employment decreases because we have higher $\hat{h}_2(\tau)$ on the left side of τ^* and lower $\hat{h}_2(\tau)$ on the right side of τ^*.

As shown in section 2, tasks with higher equipment intensities are consistent with routine-intensive tasks; hence, the proposition states that decreasing routine



(a) Assignment

(b) Changes in employment

Fig. 4.1: Equilibrium comparison: A_{1e} vs $A_{2e} > A_{1e}$

employment can be caused by an increase in the productivity of the equipment-producing sector.

The condition of sufficiently small $\alpha_h'(\tau)$ is assumed because the impact of the change in equipment prices on human capital depends on the relative size of α_h as well as on α_e. Note that this condition does not imply that the range of $\alpha_h(\tau)$ must be tight. In a numerical example with $\alpha_h(\tau)$ varying from .2 to .9, we still see changes in the assignment function consistent with the analytical comparative statics[9].

The intuition of the proposition is as follows. An increase in the equipment productivity (A_e) leads to a decrease in the price of equipment (q_e), which increases the productivity of all tasks but to a greater extent for tasks with higher equipment intensities. When the production is more complementary in the tasks than the relation between humans and technology ($\epsilon < 1$), the rise of relative productivity causes factors to flow out to other tasks, which results in polarization.[10]

The Rise of Software The profits from providing software and equipment are proportional to the demand, which, in turn, is proportional to the task output times the factor intensity of the task. Hence, changes in the relative size of task production results in changes in the profit from providing each type of capital

[9]The numerical exercise can be provided upon request.

[10]The intuition is similar to other papers in the literature, for example, Lee and Shin (2017), Goos et al. (2014), and Cheng (2017). Importantly, we highlight that the changes in the occupational structure itself can lead to another type of task-specific technological change, which we explore in the following propositions.

according to the corresponding factor intensity.

We know from proposition 1 that the employment share around τ^* (in the middle) shrinks. As long as $\alpha_h'(\tau)$ is small, the share of task production around τ^* must also decrease. Meanwhile, $\alpha_e(\tau^*) > \alpha_s(\tau^*)$, together with $\alpha_e(\bar{\tau}) < \alpha_s(\bar{\tau})$ and $\alpha_e(0) = \alpha_s(0)$ (assumption 4), imply that a decrease in production share around τ^* actually decreases e more than s, and an increase in production share around $\bar{\tau}$ increases s more than e. Therefore, providing software becomes more profitable for innovators. Innovators then focus innovation toward software, resulting in higher N_s/N_e in the new steady state.

Although this prediction is valid for most reasonable quantifications, we must impose tight restrictions on the structures of the intensities over the entire range of $\tau \in [0, \bar{\tau}]$ to prove the analytical proposition as we are comparing the ratio of two integrations over all τ ($\pi_e/\pi_s \propto \int \alpha_e(\tau)p(\tau)T(\tau)d\tau / \int \alpha_s(\tau)p(\tau)T(\tau)d\tau$). To express the analytical proposition in a simpler way, we consider an approximation with three discrete tasks ($j = 0, 1, 2$ for low, middle, and high) for now. Specifically, consider a production technology given by

$$Y = \left(\sum_j \gamma_j^{\frac{1}{\epsilon}} T_j^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}} \text{ for } j = 0, 1, 2, \tag{19}$$

with $T_j = (b(h,j)l(h))^{\alpha_{h,j}} E^{\alpha_{e,j}} S^{\alpha_{s,j}}$. The detailed derivation of the equilibrium conditions for this approximation can be found in appendix B. With this approximation, assumptions 1 and 4 are replaced by the following.

Assumption 5 (distribution-II) *The measure $\mathcal{M} : [1, \bar{h}] \to [0, 1]$ has a differentiable p.d.f. $\mu(h)$, where $\mu(h)$ is sufficiently small everywhere.*

Assumption 6 (intensities-II) *The discrete intensities satisfy the following.*

6.1 $\frac{\alpha_{e,1}}{\alpha_{h,1}} > \frac{\alpha_{e,0}}{\alpha_{h,0}} \approx \frac{\alpha_{e,2}}{\alpha_{h,2}}$.

6.2 $\alpha_{e,0} \approx \alpha_{s,0}$, $\alpha_{e,1} > \alpha_{e,2}$, *and* $\alpha_{s,2} > \alpha_{s,1}$.

In assumption 5, we add the requirement for $\mu(h)$ to be sufficiently small to consider the discretization as an approximation of continuous tasks matched with a continuum of skills.

Assumption 6.1 states that task 1 is more equipment intensive relative to labor than task 0 and task 2. Assumption 6.2 states that middle-skill tasks use equipment more than software, high-skill tasks use software more than equipment, and low-skill tasks use software and equipment similarly.

Denote the total varieties in the previous steady state as N_{s1} and N_{e1} and those in the new steady state as N_{s2} and N_{e2}. Then, we have

Proposition 2 (Rise of software) *Consider $A_{1e} < A_{2e}$ with discretized tasks (19), where equipment variety is at least as large as software variety in the original equilibrium ($N_{e1} \geq N_{s1}$). Suppose $\epsilon < 1$, $\nu_e = \nu_s$, assumptions 2, 5, and 6. In the new steady state, software variety increases more than equipment variety, i.e., $N_{s2}/N_{e2} > N_{s1}/N_{e1}$.*

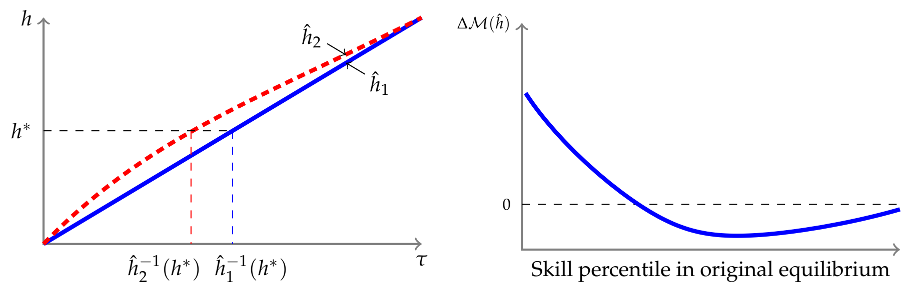
Proof *In appendix C.* ∎

Skill Demand Reversal We now show that an increase in N_s results in skill demand reversal (i.e., a decrease in the demand for high-skill labor). We consider $N_{s2} > N_{s1}$ and denote $\hat{h}_1$ and $\hat{h}_2$ as the equilibrium assignment corresponding to N_{s1} and N_{s2}, respectively.

Proposition 3 (Skill demand reversal) *Consider $N_{s2} > N_{s1}$ and suppose $\epsilon < 1$ and assumptions 1 to 4. With sufficiently small $\alpha_h'(\tau)$, the matching function shifts upward everywhere, i.e., $\hat{h}_2(\tau) > \hat{h}_1(\tau)$ for all $\tau \in (0, \bar{\tau})$.*

Proof *In appendix C.* ∎

Note that an increase in variety increases the productivity of software-intensive tasks more than that of other tasks (equation 11). Following the same intuition as in the proposition 1, this increase would lead to a reallocation of labor from high-skill tasks to lower-skill tasks under complementarity ($\epsilon < 1$). The change in the assignment function is depicted in figure 4.2, which shows that all workers downgrade their tasks.



(a) Assignment (b) Changes in employment

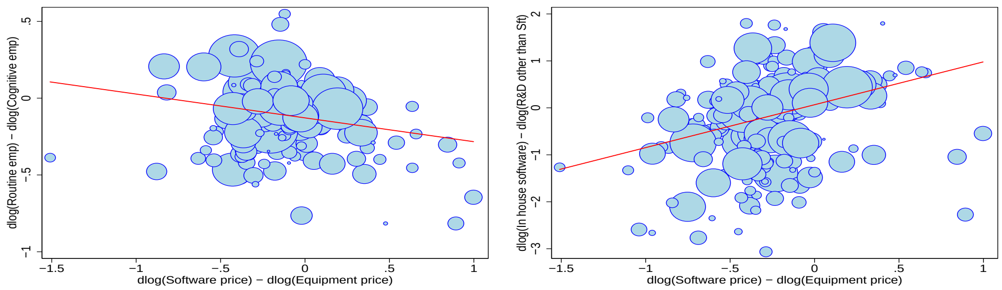
Fig. 4.2: Equilibrium comparison: N_{s1} and $N_{s2} > N_{s1}$

5 Empirical Evidence

This section checks the validity of the model's predictions using industry-level data. Specifically, we test two predictions. First, the model predicts a negative relationship between changes in the relative price of software to equipment and changes in middle-skill employment relative to high-skill employment. Second, the model implies a positive correlation between changes in the relative price of software to equipment and changes in software innovation relative to other innovation. Note that, in our model, the prices of equipment and software are inversely related to productivity in the equipment- and software-producing sectors, respectively.

We measure the relative price of equipment to software by industry from Section 2 of the Fixed Asset Table provided by BEA. The prices are different by industry as each industry uses a different combination of subitems within the category of equipment or software. For the relative employment of middle-skill to high-skill occupations, we use the employment of routine occupations divided by the employment of cognitive occupations by industry, computed from census data.[11] Finally, the relative size of software innovation to other innovation is measured

[11] Routine occupations include machine operations, office and sales, mechanics, construction and production, and transportation occupations. Cognitive occupations are management, professional, and technical occupations. The level of employment is obtained from the 1980, 1990, and 2000 censuses and the American Community Survey (ACS) 2010, retrieved from IPUMS. We made a concordance between consistent industry code `ind1990` and `indnaics` using employment in the 2000 Census. Then, employment by `indnaics` was merged into 61 BEA industry codes based on a concordance between the BEA industry code and NAICS.



(a) Relative employment vs Relative price (b) Relative innovation vs Relative price

Fig. 5.1: Changes in the relative price and employment / innovation

by own account software investment (in-house software investment by firms) divided by other R&D's by industry.

Figure 5.1a shows the differences in the growth of middle-skill and high-skill employment against changes in the price of software relative to the price of equipment. Figure 5.1b shows the changes in software innovation net of R&D expenditures excluding software against changes in the relative price. The first has a negative relation, and the second has a positive relation, consistent with the model's predictions.

To determine whether these relations are statistically significant, we estimate the following regression:

$$\Delta \log y_{i,t} = a + c_t + \Delta \log(q_{s,i,t}/q_{e,i,t}) + \varepsilon_{i,t},$$

where $y_{i,t}$ is either the ratio of routine (middle-skill) employment to cognitive (high-skill) employment or the ratio of in-house software investment to R&D expenditures excluding software. The estimation results, which show significant relations between the two variables, are given in table 5.1.

6 Quantitative Analysis

We now quantify the importance of the main mechanism. To map the tasks to occupational groups in the data (as in table 6.3), we discretize our model (appendix

Tab. 5.1: Estimation results

	Routine/Cognitive		Sft/R&D (excl. sft.)	
Sft price/ Eqp price	-0.220*** (0.000)	-0.152** (0.014)	+0.747** (0.016)	+0.717*** (0.001)
Fixed Effects	Yes	No	Yes	No
R^2	0.172	0.054	0.117	0.064

p-values in parentheses.

B).[12] In addition, we introduce exogenous task-specific productivities (M_j) in the quantitative analysis, as in equation (20). The M_j's are introduced to capture any types of routine-biased changes not captured in capital-embodied technical changes (A_i's and N_i's).

$$y_j(h) = M_j \left[\left\{ \alpha_{h,j} \left(b(h,j) l_j(h) \right)^{\frac{\sigma_e - 1}{\sigma_e}} + \alpha_{e,j} E^{\frac{\sigma_e - 1}{\sigma_e}} \right\}^{\frac{\sigma_e(\sigma_S - 1)}{(\sigma_e - 1)\sigma_S}} + \alpha_{s,j} S^{\frac{\sigma_S - 1}{\sigma_S}} \right]^{\frac{\sigma_S}{\sigma_S - 1}} \tag{20}$$

Exogenous Variation Note that we have two types of exogenous productivity (A_i's and M_j's), as well as endogenous productivity (N_i's), and the changes in exogenous productivities (A_i's and M_j's) are sources of exogenous variation in the main analysis.

We use the TFP of equipment- or software-producing industries to map A_i's as data[13]. The additional routine-biased technical changes (M_j's corresponding to routine jobs – administrative, machine operation, transportation, sales, mechanical, and production occupations) are set to match changes in the employment share of routine occupations in the model with the data exactly.

[12] Following the literature, we label high-, middle-, and low-skill occupations as cognitive, routine, and manual occupations, respectively. Cognitive occupations include management, professional, and technical occupations. Routine occupations are administrative, machine operation, transportation, sales, mechanical, and production occupations. Finally, manual occupations are low-skill services occupations.

[13] In the model, increases in A_e and A_s result in a decline in the price of equipment and software, whereas changes in N_e and N_s do not alter the price of capital. Indeed, the price of equipment decreases more quickly than that of software, and the TFP of equipment-producing industries increases faster than that of software. In contrast, software development expenditures rise more quickly than other types of R&D.

Taking A_i's as exogenous, we do not answer why the production of equipment has been more productive since 1980 to begin with. The increase in the productivity of equipment production mostly reflects the rise of the speed of computing processors (i.e. Moore's Law), and we take Moore's Law as given. We then ask how this initial innovation affects the occupational structure *and* the following direction of technical change, represented by increasing varieties of equipment or software (N_i's).

The changes in A_i's themselves are task-biased as intensities of equipment and software are different across occupational groups. However, it does not capture "all" the routine-biased forces that have happened since 1980. For example, increasing offshoring or trade could also contribute to the decline in routine employment. The exogenous task-specific productivities (M_j's) reflect components of the potential routine-biased forces other than capital-embodied technical changes. A natural question is how much of the changes in the routine employment can be captured only through A_i's quantitatively. We answer this question in one of the exercises.

Scenarios We perform two main exercises. The first is to investigate the extent to which the endogenous software innovation channel explains the rise of software and skill demand reversal in the data. For this exercise, we match the changes in the employment share of middle-skill occupations with A_i's and M_j's and consider the employment dynamics of high- and low-skill occupations generated from the models with innovation and that without innovation.

The next exercise aims to understand the extent to which changes in the productivity of the equipment- and software-producing sectors only, excluding M_j's, account for the shifts in the employment share between occupations. To address this question, we repeat the simulation with all the other parameters fixed, assuming a constant M_j for all routine occupations.

6.1 Calibration

We calibrate most of the parameters according to the 1980 data assuming a steady state. For the functional forms, we set the productivity function $b(h, j)$ as

$$b(h,j) = \begin{cases} \bar{h} & \text{if } j = 0 \\ h - \chi_j & \text{if } j \geq 1 \end{cases}$$

and the skill distribution $\mathcal{M}(h)$ as

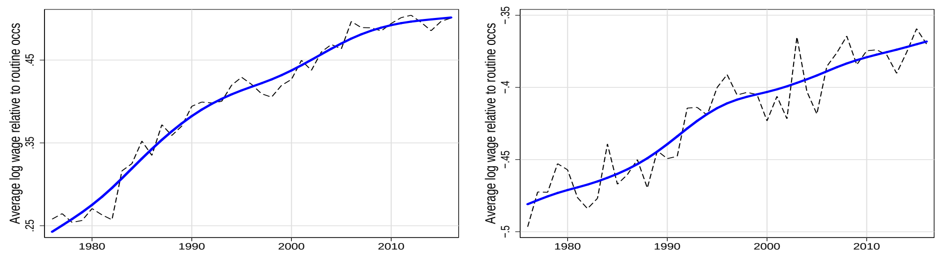
$$\mathcal{M}(h) = 1 - h^{-a}.$$

Share parameters The weight parameters in the final production (γ_j's) are taken from the employment share by occupation in 1980. The χ_j's and a are determined to match the payroll share across occupational groups in 1980. Between-factor intensities by task ($\alpha_h,\ \alpha_e,\ \alpha_s$) are matched to equipment and software investment by occupational group obtained in section 2.4, and labor share in 1980. For the benchmark analysis, we map the equipment in the model to industrial equipment in the data because it has the closest relation with the routineness of occupations (figure 2.4).

The elasticity of substitution between tasks (ϵ) We set ϵ to minimize the root mean squared error of the changes in payroll share between 1980 and 2010 by occupation. That is, we repeat the calibration procedure to minimize (21) with varying ϵ, where ω_τ is the payroll share of occupational group j.

$$\left[\sum_{j=1}^{J} \left[(w^m_{j,2010} - w^m_{j,1980}) - (w^d_{j,2010} - w^d_{j,1980}) \right]^2 / J \right]^{\frac{1}{2}} \tag{21}$$

Intuitively, occupations are complementary when changes in quantity (employment share) and changes in the relative price (relative wage) move in the same direction, which is actually the case as depicted in figure 2.3 and 6.1. The resulting parameter value is 0.302, confirming the complementarity.



(a) Cognitive to Routine

(b) Manual to Routine

Fig. 6.1: Log of relative wage

The elasticity of substitution between labor and capital For σ_e and σ_s, we match linear trends of the aggregate labor share and labor share only with equipment capital[14] Note that the labor share with only equipment capital in a given task τ is given by $LS_{-s} = \frac{wL}{wL+p_e\tilde{E}} = 1/\left[1+\left(\frac{\alpha_e}{\alpha_h}\right)^{\sigma_e}\left(\nu_e A_e N_e^{\varphi_e}\omega\right)^{\sigma_e-1}\right]$, which does not depend on σ_s.

The fact that the aggregate labor share and labor share with equipment capital alone show different trends – the former downward and the latter upward – makes this strategy even more useful (figure 6.4a).

The markups We estimate the markup-related parameters ν_e and ν_s using the Industry Account and Fixed Asset Table from BEA, following Domowitz et al. (1988). Specifically, we estimate

$$\Delta \log q_{it} - \alpha_{Lit}\Delta \log l_{it} - \alpha_{mit}\Delta \log m_{it} = c_i + b\Delta \log q_{it} + \varepsilon_{it},$$

where q is gross output/capital, l is employment/capital, m is intermediate input/capital, and α_{Lit} and α_{Mit} are the labor and intermediate shares, respectively. The estimation results are presented in table 6.1.

[14] We compute the labor share with equipment capital only following Koh et al. (2016). Specifically, a standard asset pricing formula gives $R_i = (1+r)q_i - q_i'(1-\delta_i)$, where R_i is the gross return on capital type i, q_i is the relative price of capital type i, δ_i is a depreciation rate of capital type i, and r is the net rate of return. Assuming the CRS production technology (where one minus the labor share is equal to $\sum_i R_i K_i/Y$), we impute the gross rate of return on equipment, R_e. The labor share with equipment capital only can be computed by $CE/(CE + R_e K_e)$, where CE is the compensation of employees.

Tab. 6.1: Estimation results: markup

	Equipment[1)]	Software[2)]
b	.228	.473
	(.000)	(.113)
N	333	37

Note: 1) Industries 331, 332, 333, 334, 335, 3361MV, 3364TO, 337, and 339. 2) Industry 511. 3) p-values in parentheses.

Changes in A_i's and M_j's We assume the economy was in a steady state in 1980 and compute a new steady state corresponding to the exogenous changes (A_e, A_s, M_j's). For 1980, we set $A_i = M_j = 1$. The A_i's are set to equal the TFPs of the equipment- and software-production sectors, computed in section 2.1. For routine occupations ($j = 2, 3, 4, 5, 7, 8$[15]), we vary M_j's to fit the employment dynamics of routine occupations in the calibration procedure. Again, we compare a scenario allowing for an endogenous response of innovators (with changing N_e and N_s) with a scenario without endogenous innovation (with fixed N_e and N_s). To make the two scenarios compatible, we recalibrate M_j's to match the employment dynamics of routine occupations in both scenarios. Both scenarios therefore produce different dynamics of cognitive and manual occupations only, which are never targeted in the calibration procedure. The exogenous productivities are shown in table 6.2.

Tables 6.2, 6.3, and 6.4 summarize all the calibration results. A detailed description of the calibration procedure is provided in appendix D.

6.2 Simulation Results

The Pattern of Occupational Employment Figure 6.2 shows the decadal pattern from 1980 to 2010. The deviation from the initial trend in cognitive occupation in the model captures 75% of the actual deviation in the data (figure 6.2a), and the deviation from the initial trend in manual employment in the model is 70% of that in the data (figure 6.2b). The model captures not only the magni-

[15] These are administrative, machine operation, transportation, sales, mechanical, and production occupations.

Tab. 6.2: Exogenous productivities

	1980	1990	2000	2010	Obtained from
A_e, TFP of equipment production	1.00	1.42	2.45	3.80	Data
A_s, TFP of software production	1.00	0.68	0.75	1.13	Data
M_j's with endogenous N_i's					
Administrative (j=2)	1.00	1.21	1.32	0.92	Routine
Machine operators (j=3)	1.00	1.33	1.58	2.51	employment
Transportation (j=4)	1.00	1.10	1.13	1.15	dynamics
Sales (j=5)	1.00	0.92	1.06	0.84	(with
Mechanics (j=7)	1.00	0.94	0.76	0.84	varying N_i's)
Production (j=8)	1.00	1.49	1.30	1.87	
M_j's with fixed N_i's					
Administrative (j=2)	1.00	1.25	1.53	1.78	Routine
Machine operators (j=3)	1.00	1.34	1.59	2.15	employment
Transportation (j=4)	1.00	1.11	1.13	0.98	dynamics
Sales (j=5)	1.00	0.94	1.16	1.25	(with
Mechanics (j=7)	1.00	0.95	0.77	0.75	fixed N_i's)
Production (j=8)	1.00	1.51	1.31	1.73	

Tab. 6.3: Parameters by occupation

	α_e	α_s	α_h	γ	χ
Low-skilled services (j=1)	0.185	0.007	0.808	0.004	
Administrative (j=2)	0.062	0.152	0.786	0.741	0.000
Machine operators (j=3)	0.639	0.013	0.348	0.069	0.002
Transportation (j=4)	0.545	0.012	0.443	0.030	0.027
Sales (j=5)	0.083	0.012	0.905	0.002	0.029
Technicians (j=6)	0.262	0.014	0.724	0.002	0.071
Mechanics (j=7)	0.691	0.015	0.294	0.129	0.071
Production (j=8)	0.523	0.016	0.461	0.018	0.096
Professionals (j=9)	0.132	0.008	0.860	0.003	0.097
Management (j=10)	0.019	0.008	0.973	0.002	0.097
Target	Equipment, software, and labor share			Employment	Payroll

tude but also the timing of the changes in the trends, as it produces much larger changes during 2000-2010 than during the first two decades. Without endogenous innovation, the simulation generates almost no variation in the trends of high- and low-skill employment.

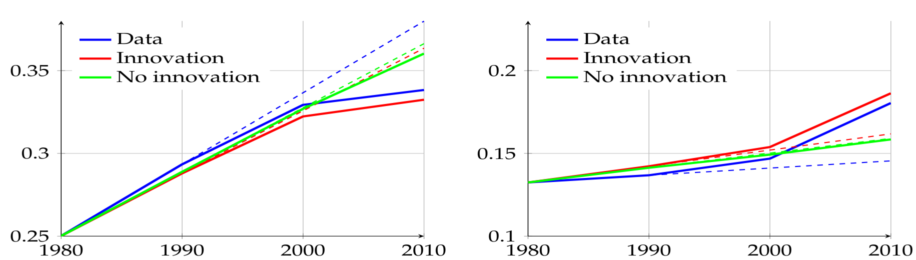
Tab. 6.4: Remaining parameters

	Value	Obtained from
σ_s	1.425	Labor share with and without software in 2010
σ_e	0.981	
ν_e	0.772	Estimation (table 6.1)
ν_s	0.527	
ϵ	0.302	Changes in average wage by occupation

The Rise of Software The ratio of software investment to industrial equipment investment increases from 0.16 to 1.7 in the data, an increase of more than tenfold. Since we match the initial level of relative investment 0.16 exactly by calibration, we compare the level of the ratio in 2010 to determine how well the model explains the rise of software. The full model with innovation explains 63% of the increase in software investment relative to that in equipment (figure 6.3a). If we remove the endogenous innovation channel (i.e., no changes in N_s and N_e), the model generates only 19% of the change in the ratio of software to equipment (green line).

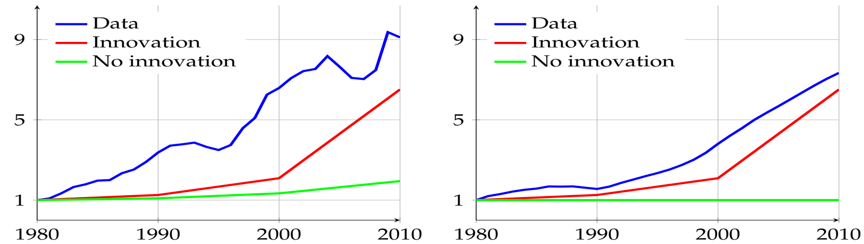
Figure 6.3b shows the ratio of software variety to equipment variety (N_s/N_e), measured by cumulative R&D expenditures. Both the data and model show an increasing pattern, with the model explaining 89% of that in the data.

The Decline of Labor Share We use the labor share trend as a target variable to calibrate the elasticity of substitution (σ_e and σ_s); therefore, it is not surprising that the labor share in the model exactly matches the labor share trend in the



(a) Cognitive employment share (b) Manual employment share

Fig. 6.2: Simulation results – employment shares by occupation

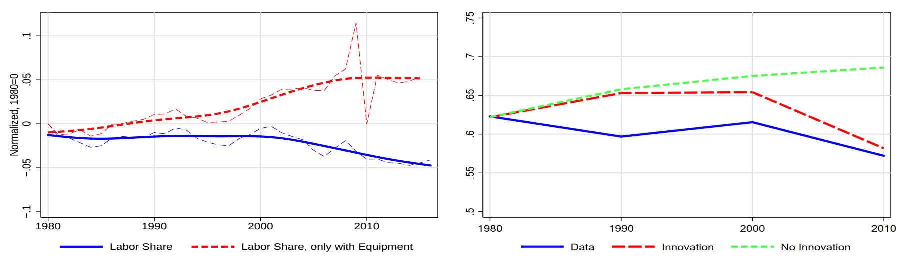


(a) Software / Equipment Investment (b) Software / Equipment Variety

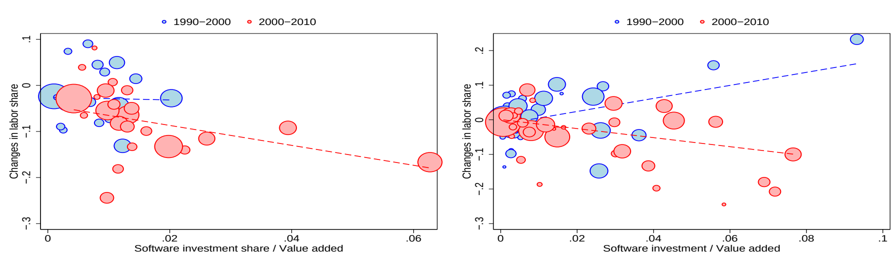
Fig. 6.3: Simulation results – investment and innovation (1980=1)

data. What is interesting is that the simulation without endogenous software innovation produces an almost flat labor share (figure 6.4b).

This result occurs because the elasticity of substitution between equipment and labor (σ_e) is close to one; hence, exogenous variation – mostly equipment related – does not generate a declining labor share. Therefore, the declining labor



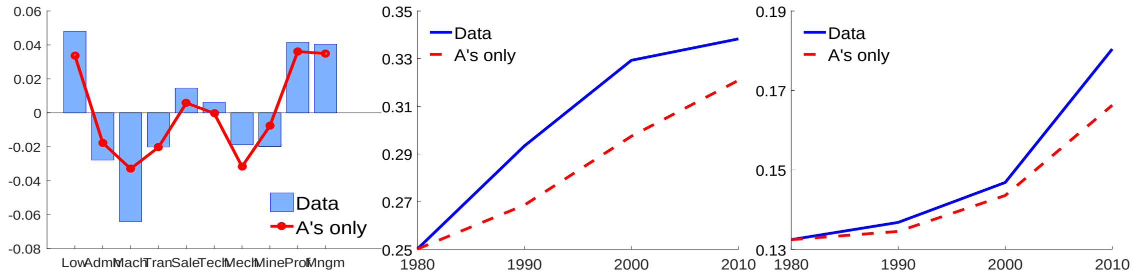
(a) Labor share trend[1)] (b) Labor share simulation



(c) By industry – Manufacturing (d) By industry – Services[2)]

Fig. 6.4: Labor share and software

Note: 1) The labor share only with equipment capital is constructed following Koh et al. (2016). The solid lines are HP trend with smoothing parameter 100. They are normalized to 0 in 1980. 2) Industry 514 (with changes in labor share greater than 1 in both periods) has been excluded from this figure.



(a) By occupation, 1980-2010 (b) Cognitive employment (c) Manual employment

Fig. 6.5: Simulation results: with constant M_j's

share in our model is mostly a result of endogenously increasing software investment. We highlight a negative correlation between software investment and the labor share not only across time (figure 6.4a) but also across industries, especially since 2000 (figures 6.4c and 6.4d).

6.2.1 Changes in A_i's only with constant M_j's

Now we investigate how much of the observed changes in the sectoral productivities alone explain the variation in the share of employment by occupation.

Overall, the changes in A_i's explain 80%, 77%, and 71% of the changes in cognitive, routine and manual employment, respectively (figure 6.5). Two features are worth noting.

First, all ten occupational groups move in the same direction as the data (figure 6.5a), meaning that the differential growth of sectoral productivities – together with differences in the use of capital – captures the job polarization quite well. The analysis suggests that differential productivity growth on the sector level could be an underlying source of routine-biased technological change.

Second, the decadal pattern of changes in occupational employment is similar to that of the data, even without additional task-specific technical change (figure 6.5b and 6.5c). Moreover, changes in TFP generate 77% of the decline in routine occupations. We conclude that the evolution of the productivities embodied in equipment and software has been crucial in generating a pattern that is consistent with the data.

7 Conclusion

We provided a model with heterogenous tasks and two types of capital whose varieties are determined endogenously. We showed both analytically and quantitatively that the mechanism in the model is important for understanding the impact of capital-augmented technical change on the occupational structure.

An important implication is that two types of capital – software and equipment– measured in National Accounts provide a good proxy for recent technological changes. Understanding the impact of a technical change on the economy has always been an important topic, but one of the main difficulties is that technological change is not easy to measure, especially in aggregate analyses. This paper shows that the investigation of different types of capital can be a meaningful process to capture recent technological changes.

Our paper also implies that a technological change affecting a small group of occupations leads to other types of innovation, eventually affecting a broader set of occupations. Hence, the relation between occupational structure and technical change is bi-directional. Importantly, the interaction can explain both the decline in routine jobs (from 1980 to recently) and the stagnation of the growth of cognitive jobs with a greater increase in manual jobs since 2000s.

Our model has many useful extensions that can be implemented easily. For example, further decomposition of equipment capital into subcategories would be helpful for understanding more detailed changes in the occupational structure through technological changes embodied in capital. Further, integrating a multi-sector structure would provide interesting implications with respect to the relation between polarization and structural changes and the evolution of task-specific and sector-specific productivity, as in Aum et al. (2018).

Though not as straightforward, the analysis herein could also lead to many interesting future research topics. For example, by using firm-level software and equipment investment data, we could generate interesting implications regarding the impact of technological change on firm-level heterogeneity and occupation-level heterogeneity. Many countries are attempting to broaden the types of capital measured in National Accounts, and a multi-country extension would also be meaningful, enabling the analysis of trade or offshoring as well as technological changes. We have also experienced changes in the skill composition of workers in recent decades. Analyzing the upskilling or deskilling of certain tasks with the use of capital could also be an interesting topic to explore.

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Appendices

A Use of Equipment and Software by Occupation

The data on capital use by occupation are constructed by combining the BEA NIPA and O*NET Tools and Technology Database. In NIPA table 5.5, the investment in non-residential equipment is categorized into 25 types. In UNSPSC, the classification system used in the O*NET Tools and Technology database, there are 4,300 commodities in 825 classes, in 173 families, and in 36 segments.

To construct a mapping between the two databases, we first assign one of the NIPA investment types to the relevant segment in UNSPSC. Often, it is apparent that a segment includes several types of equipment investment in NIPA. In this case, we use the family categories in the assignment procedure. Again, if a family apparently includes several types in NIPA, we use classes. Through this procedure, we could make a rough concordance between a subset of UNSPSC and the types of equipment investment in NIPA. The constructed concordance is shown in table A1.

Next, we assume that two tools have the same price if they are classified in the same category. For example, the "metal cutting machines" category in UNSPSC is assigned to the "metalworking machinery" NIPA investment type. The value of using the metal cutting machines is then the amount of investment in metalworking machinery divided by total use of all the commodities in the metalworking machinery category, where the total use of all the tools in metalworking machinery is defined as sum of a number of total employment of each occupation times a number of UNSPSC commodities assigned to the metalworking machinery that each occupation uses.

The method is assuming that the number of tools above well represent the value of the tools only within the NIPA investment category. Across the NIPA investment categories, each number of tools used would be given a different weight, according to the average amount of investment given to each tool. The procedure may make a big difference from average number of tools if a category with many commodities had small values compared to a category with few commodities. However, as more differentiated categories are usually advanced (and, hence, have expensive items), we expect not much of a difference from the adjustment.

B Discrete Approximation of the Model

This section discusses equilibrium conditions with discrete approximation of the model. For the approximation, assumption 1 and 4 are replaced by assumption 5 and 6 in section 3 and 4.

The task production is given by equation (19) with tasks discretized into $j = 0, 1, \cdots, J$. Now, the tasks are discrete, so workers are sorted into each task according to cutoff level of human

Tab. A1: Concordance between NIPA equipment investment types and UNSPSC

NIPA		UNSPSC	
Line	Title	Code	Title
3	**Information processing equipment**		
4	Computers and peripheral equipment	43210000	Computer Equipment and Accessories
5	Communication equipment	43190000, 45110000	Communications Devices and Accessories, Audio and visual presentation and composing equipment
6	Medical equipment and instruments	42000000	Medical Equipment and Accessories and Supplies
9	Non-medical instruments	41000000	Laboratory and Measuring and Observing and Testing Equipment
10	Photocopy and related equipment	45100000, 45120000	Printing and publishing equipment, Photographic or filming or video equipment
11	Office and accounting equipment	44100000, 31240000	Office machines and their supplies and accessories, Industrial optics
12	**Industrial equipment**		
13	Fabricated metal products	27000000, 31150000, 31160000, 31170000, 40140000, 40170000	Tools and General Machinery, Rope and chain and cable and wire and strap, Hardware, Bearings and bushings and wheels and gears, Fluid and gas distribution, Pipe piping and pipe fittings
14	Engines and turbines	26101500, 26101700	Engines, Engine components and accessories
17	Metalworking machinery	23240000, 23250000, 23260000, 23270000, 23280000	Metal cutting machinery and accessories, Metal forming machinery and accessories, Rapid prototyping machinery and accessories, Welding and soldering and brazing machinery and accessories and supplies, Metal treatment machinery
18 + 19	Special industry machinery, n.e.c. + General industrial, including materials handling, equipment	23100000, 23110000, 23120000, 23130000, 23140000, 23150000, 23160000, 23180000, 23190000, 23200000, 23210000, 23220000, 23230000, 23290000, 24100000, 24110000, 31140000, 40000000	Raw materials processing machinery, Petroleum processing machinery, Textile and fabric machinery and accessories, Lapidary machinery and equipment, Leatherworking repairing machinery and equipment, Industrial process machinery and equipment and supplies, Foundry machines and equipment and supplies, Industrial food and beverage equipment, Mixers and their parts and accessories, Mass transfer equipment, Electronic manufacturing machinery and equipment and accessories, Chicken processing machinery and equipment, Sawmilling and lumber processing machinery and equipment, Industrial machine tools, Material handling machinery and equipment, Containers and storage, Moldings, Distribution and Conditioning Systems and Equipment and Components
20 + 41	Electrical transmission, distribution, and industrial apparatus + Electrical equipment, n.e.c.	26101100, 26101200, 26101300, 26110000, 26120000, 26130000, 26140000, 39000000	Electric alternating current AC motors, Electric direct current DC motors, Non-electric motors, Batteries and generators and kinetic power transmission, Electrical wire and cable and harness, Power generation, Atomic and nuclear energy machinery and equipment, Electrical Systems and Lighting and Components and Accessories and Supplies
21	**Transportation equipment**		
22 + 25	Trucks, buses, and truck trailers + Autos	25100000	Motor vehicles
26	Aircraft	25130000	Aircraft
27	Ships and boats	25110000	Marine transport
28	Railroad equipment	25120000	Railway and tramway machinery and equipment
29	**Other equipment**		
30	Furniture and fixtures	56000000	Furniture and Furnishings
33	Agricultural machinery	21000000	Farming and Fishing and Forestry and Wildlife Machinery and Accessories
36	Construction machinery	22000000	Building and Construction Machinery and Accessories
39	Mining and oilfield machinery	20000000	Mining and Well Drilling Machinery and Accessories
40	Service industry machinery	48000000	Service Industry Machinery and Equipment and Supplies

capital $\hat{h}_j$. More precisely, we have a sequence of human capital $\{\hat{h}_j\}_{j=0,\cdots,J+1}$ such that a worker with $h \in [\hat{h}_j, \hat{h}_{j+1})$ is sorted into task j with $\hat{h}_0 = \underline{h}$ and $\hat{h}_{J+1} = \bar{h}$.

A worker with exactly the threshold level of human capital should be indifferent between tasks so that

$$\omega_j b(\hat{h}_j, j) = \omega_{j-1} b(\hat{h}_j, j-1), \text{ for all } j, \text{ for } , j = 1, \cdots, J \tag{B.1}$$

replacing the original equilibrium condition (9).

The task production solves

$$\max p_j T_j - \int_h w(h) l(h) dh - \int_{k=0}^{N_e} p_e(k) e(k) dk - \int_{k=0}^{N_s} p_s(k) s(k) dk,$$

which gives the FOCs,

$$\begin{aligned} w(h) &= \omega_j b(h,j) = p_j T_j^{\frac{1}{\sigma_s}} H_j^{\frac{1}{\sigma_e} - \frac{1}{\sigma_s}} \left(\int_{\hat{h}_j}^{\hat{h}_{j+1}} b(h,j) \mu(h) dh \right)^{-\frac{1}{\sigma_e}} b(h,j), \\ \frac{1}{A_e \nu_e} &= p_j T_j^{\frac{1}{\sigma_s}} H_j^{\frac{1}{\sigma_e} - \frac{1}{\sigma_s}} (N_e)^{\frac{\sigma_e - 1}{\sigma_e \nu_e} - 1} e_j^{-\frac{1}{\sigma_e}}, \\ \frac{1}{A_s \nu_s} &= p_j T_j^{\frac{1}{\sigma_s}} (N_s)^{\frac{\sigma_s - 1}{\sigma_s \nu_s} - 1} s_j^{-\frac{1}{\sigma_s}}, \end{aligned}$$

using the fact that $p_e = 1/(A_e \nu_e)$, $p_s = 1/(A_s \nu_s)$, $e_j(k) = e_j$, and $s_j(k) = s_j$ in equilibrium, and $H_j := \left[\alpha_{h,j} (\int_{\hat{h}_j}^{\hat{h}_{j+1}} b(h,j) \mu(h) dh)^{\frac{\sigma_e - 1}{\sigma_e}} + \alpha_{e,j} (\int_{k=0}^{N_e} e(k)^{\nu_e} dk)^{\frac{\sigma_e - 1}{\sigma_e \nu_e}} \right]^{\frac{\sigma_e}{\sigma_e - 1}}$.

Combining the FOCs, we obtain

$$p_j = \left[\left(\alpha_{h,j}^{\sigma_e} \omega_j^{1-\sigma_e} + \alpha_{e,j}^{\sigma_e} \left(\nu_e A_e N_e^{\varphi_e} \right)^{\sigma_e - 1} \right)^{\frac{1-\sigma_s}{1-\sigma_e}} + \alpha_{s,j}^{\sigma_s} \left(\nu_s A_s N_s^{\varphi_s} \right)^{\sigma_s - 1} \right]^{\frac{1}{1-\sigma_s}}, \text{ for } j = 0, \cdots, J \tag{B.2}$$

which replaces equation (11).

The demand for each task is from

$$\max Y - \sum_j p_j T_j,$$

which gives

$$p_j = \left(\frac{\gamma_j Y}{T_j} \right)^{\frac{1}{\epsilon}}.$$

Combining this with FOCs, we obtain

$$p_j^{\epsilon-\sigma_s} = \frac{\gamma_j \alpha_{h,j}^{\sigma_s} \left(\alpha_{h,j}^{\sigma_e} \omega_j^{1-\sigma_e} + \alpha_{e,j}^{\sigma_e} \left(\nu_e A_e N_e^{\varphi_e} \right)^{\sigma_e - 1} \right)^{\frac{\sigma_e - \sigma_s}{1-\sigma_e}} Y}{\omega_j^{\sigma_e} \int_{\hat{h}_j}^{\hat{h}_{j+1}} b(h,j)\mu(h)dh}, \text{ for } j = 0, \cdots, J, \tag{B.3}$$

replacing equation (10).

Now, the equilibrium thresholds $\hat{h}_j$'s, wage rate ω_j's and prices p_j's are obtained by solving equation (B.1) to (B.3), which are $3J+1$ equations with the same number of unknowns.

C Proof

Proof of lemma 2 *Since assignment function $\hat{h}(\tau)$ is strictly increasing, its inverse $\hat{\tau}(h)$ is well defined. From the demand for tasks, equation (7), we know that there will be a strictly positive task output $T(\tau) > 0$ (and, hence, $l(h, \hat{\tau}(h)) > 0$) for all $\tau \in [0, \bar{\tau}]$. The equation (8) and lemma 1 then implies*

$$w(h) = \omega(\hat{\tau}(h))b(h, \hat{\tau}(h)) \geq \omega(\hat{\tau}(h'))b(h, \hat{\tau}(h')), \text{ and}$$
$$w(h') = \omega(\hat{\tau}(h'))b(h', \hat{\tau}(h')) \geq \omega(\hat{\tau}(h))b(h', \hat{\tau}(h)).$$

Combining these two inequalities, we have

$$\frac{b(h, \hat{\tau}(h'))}{b(h, \hat{\tau}(h))} \leq \frac{\omega(\hat{\tau}(h))}{\omega(\hat{\tau}(h'))} \leq \frac{b(h', \hat{\tau}(h'))}{b(h', \hat{\tau}(h))}$$

Let $\tau = \hat{\tau}(h)$ and $\tau' = \hat{\tau}(h')$. Since $\hat{\tau}$ has an inverse function $\hat{h}$, the above inequality is equivalent to

$$\frac{b(\hat{h}(\tau), \tau')}{b(\hat{h}(\tau), \tau)} \leq \frac{\omega(\tau)}{\omega(\tau')} \leq \frac{b(\hat{h}(\tau'), \tau')}{b(\hat{h}(\tau'), \tau)}$$

By taking the log on both sides and dividing by $\tau' - \tau$,

$$\frac{\log b(\hat{h}(\tau), \tau') - \log b(\hat{h}(\tau), \tau)}{\tau' - \tau} \leq \frac{-(\log \omega(\tau') - \log \omega(\tau))}{\tau' - \tau} \leq \frac{\log b(\hat{h}(\tau'), \tau') - \log b(\hat{h}(\tau'), \tau)}{\tau' - \tau}$$

As $\tau' - \tau \to 0$, we have

$$\frac{d \log \omega(\tau)}{d\tau} = -\frac{\partial \log b(\hat{h}(\tau), \tau)}{\partial \tau},$$

which is the equation (9).

Now, consider the task production. For notational convenience, we introduce

$$H(h,\tau) = \left[\alpha_h(\tau)(b(h,\tau)l(h))^{\frac{\sigma_e-1}{\sigma_e}} + \alpha_e(\tau)\left(\int_0^{N_e} e(k,\tau)^{\nu_e} dk\right)^{\frac{\sigma_e-1}{\sigma_e\nu_e}}\right]^{\frac{\sigma_e}{\sigma_e-1}}$$

From

$$\max\ p(\tau)T(\tau) - \int_h w(h)l(h,\tau)dh - \int_0^{N_s} p_s(k)s(k,\tau)dk - \int_0^{N_e} p_e(k)e(k,\tau)dk,$$

we have the following first-order conditions:

$$w(h) \geq \alpha_h(\tau)p(\tau)T(\tau)^{\frac{1}{\sigma_s}} H(h,\tau)^{\frac{\sigma_s-\sigma_e}{\sigma_e\sigma_s}} l(h)^{-\frac{1}{\sigma_e}} b(h,\tau), \tag{C.1}$$

$$p_e(k) = \alpha_e(\tau)p(\tau)T(\tau)^{\frac{1}{\sigma_s}} H(h,\tau)^{\frac{\sigma_s-\sigma_e}{\sigma_e\sigma_s}} \left(\int_0^{N_e} e(k,\tau)^{\nu_e}\right)^{\frac{\sigma_e-1-\nu_e\sigma_e}{\nu_e\sigma_e}} e(k,\tau)^{\nu_e-1}, \tag{C.2}$$

$$p_s(k) = \alpha_s(\tau)p(\tau)T(\tau)^{\frac{1}{\sigma_s}} \int_0^{N_s} s(k,\tau)^{\nu_s} dk^{\frac{\sigma_s-1-\nu_s\sigma_s}{\nu_s\sigma_s}} s(k,\tau)^{\nu_s-1}, \tag{C.3}$$

In the equipment- and software-producing sector, we solve

$$\max p_e(k)e(k) - e(k)/A_e, \quad \max p_s(k)s(k) - s(k)/A_s$$

subject to (C.2) *and* (C.3). *The solution gives*

$$p_e = 1/(\nu_e A_e), \quad p_s = 1/(\nu_s A_s) \quad \textit{for all } k. \tag{C.4}$$

Substituting (C.4) *into the FOCs, we obtain*

$$p(\tau) = \left[\left\{\alpha_h(\tau)^{\sigma_e}\omega(\tau)^{1-\sigma_e} + \alpha_e(\tau)^{\sigma_e}\left(\nu_e A_e N_e^{\varphi_e}\right)^{\sigma_e-1}\right\}^{\frac{1-\sigma_s}{1-\sigma_e}} + \alpha_s(\tau)^{\sigma_s}\left(\nu_s A_s N_s^{\varphi_s}\right)^{\sigma_s-1}\right]^{\frac{1}{1-\sigma_s}},$$

by combining the FOCs, which is the equation (11).

Again, from equations (C.1) *to* (C.3), *the task production* $T(\tau)$ *can be expressed by*

$$T(\tau) = p(\tau)^{-\sigma_s}\omega(\tau)^{\sigma_e}\alpha_h(\tau)^{-\sigma_e}\left(\alpha_h(\tau)^{\sigma_e}\omega(\tau)^{1-\sigma_e} + \alpha_e(\tau)^{\sigma_e}\left(\nu_e A_e N_e^{\varphi_e}\right)^{\sigma_e-1}\right)^{\frac{\sigma_s-\sigma_e}{1-\sigma_e}} \int_h b(h,\tau)l(h,\tau)dh \tag{C.5}$$

From the labor market clearing condition and lemma 1, we have

$$l(h,\tau) = \mu(h)\delta[\tau - \hat{\tau}(h)],$$

where δ is a Dirac delta function. Then, we have

$$\int_h b(h,\tau)l(h,\tau)dh = \int_{\tau'} b(\hat{h}(\tau'),\tau)\mu(\hat{h}(\tau))\delta[\tau-\tau']\hat{h}'(\tau')d\tau' = b(\hat{h}(\tau),\tau)\mu(\hat{h}(\tau))\hat{h}'(\tau).$$

Combining this with equation (7) and (C.5), we have

$$\hat{h}'(\tau) = \frac{\gamma(\tau)p(\tau)^{\sigma_s-\epsilon}\alpha_h(\tau)^{\sigma_s}\left(\alpha_h(\tau)^{\sigma_e}\omega(\tau)^{1-\sigma_e}+\alpha_e(\tau)^{\sigma_e}\left(\nu_e A_e N_e^{\varphi_e}\right)^{\sigma_e-1}\right)^{\frac{\sigma_e-\sigma_s}{1-\sigma_e}}Y}{\omega(\tau)^{\sigma_e}b(\hat{h}(\tau),\tau)\mu(\hat{h}(\tau))},$$

which is the equation (10). ∎

Proof of lemma 3 *In steady state, if it exists, $r = \pi_s/\eta_s = \pi_e/\eta_e = \rho$ from the Euler equation (17). Then $\dot{X}/X = 0$ for $X = C$, E, S, N_e, and N_s follow from usual argument. What we need to show is that there exist N_s and N_e that satisfy $\pi_s/\eta_s = \pi_e/\eta_e = \rho$.*

We begin with the following lemma.

Lemma 4 *Fix $p(\tau)$ and $\hat{h}(\tau)$. There exists a pair $(\nu_s,\nu_e) \in (0,1)\times(0,1)$ such that $s(\tau)$ is strictly decreasing in N_s and $e(\tau)$ is strictly decreasing in N_e.*

Proof Combining equation (C.1) and (C.3) (FOCs), we have

$$\begin{aligned}
s(\tau) &= N_s^{-1}N_s^{\varphi_s(\sigma_s-1)}\left(\nu_s A_s\right)^{\sigma_s}\alpha_s(\tau)^{\sigma_s}\alpha_h(\tau)^{-\frac{\sigma_e}{1-\sigma_e}}b(\hat{h}(\tau),\tau)\mu(\hat{h}(\tau))\hat{h}'(\tau)\\
&\times\left[\left(p(\tau)^{1-\sigma_s}-\alpha_s(\tau)^{\sigma_s}\left(\nu_s A_s N_s^{\varphi_s}\right)^{\sigma_s-1}\right)^{\frac{1-\sigma_e}{1-\sigma_s}}-\alpha_e(\tau)^{\sigma_e}\left(\nu_e A_e (N_s n_{es})^{\varphi_e}\right)^{\sigma_e-1}\right]^{\frac{\sigma_e}{1-\sigma_e}}\\
&\times\left(p(\tau)^{1-\sigma_s}-\alpha_s(\tau)^{\sigma_s}\left(\nu_s A_s N_s^{\varphi_s}\right)^{\sigma_s-1}\right)^{\frac{\sigma_s-\sigma_e}{1-\sigma_s}},
\end{aligned}\tag{C.6}$$

and

$$\begin{aligned}
e(\tau) &= N_e^{-1}N_e^{\varphi_e(\sigma_e-1)}\left(\nu_e A_e\right)^{\sigma_e}\alpha_e(\tau)^{\sigma_e}\alpha_h(\tau)^{-\frac{\sigma_e}{1-\sigma_e}}b(\hat{h}(\tau),\tau)\mu(\hat{h}(\tau))\hat{h}'(\tau)\\
&\times\left[\left(p(\tau)^{1-\sigma_s}-\alpha_s(\tau)^{\sigma_s}\left(\nu_s A_s (N_e/n_{es})^{\varphi_s}\right)^{\sigma_s-1}\right)^{\frac{1-\sigma_e}{1-\sigma_s}}-\alpha_e(\tau)^{\sigma_e}\left(\nu_e A_e N_e^{\varphi_e}\right)^{\sigma_e-1}\right]^{\frac{\sigma_e}{1-\sigma_e}},
\end{aligned}\tag{C.7}$$

where $n_{es} := N_e/N_s$.

From equation (C.6) and (C.7), we can express

$$\frac{\partial \log s(\tau)}{\partial N_s} = -\frac{1}{N_s} + s_1(\tau;\varphi_s), \tag{C.8}$$

$$\frac{\partial \log e(\tau)}{\partial N_e} = -\frac{1}{N_e} + e_1(\tau;\varphi_e), \tag{C.9}$$

and it is straightforward to check that $\lim_{\varphi_s \downarrow 0} |s_1(\tau; \varphi_s)| = 0$, $\lim_{\varphi_e \downarrow 0} |e_1(\tau; \varphi_e)| = 0$, and $\partial s_1 / \partial \varphi_s > 0$, $\partial e_1 / \partial \varphi_e > 0$. This expression implies that there should be $0 < \nu_s < 1$ and $0 < \nu_e < 1$, which make $s(\tau)$ strictly decreasing in N_s and $e(\tau)$ strictly decreasing in N_e. ∎

Lemma 5 *Fix $p(\tau)$ and $\hat{h}(\tau)$. With ν_e and ν_s close to one, we have the following:*

$$\lim_{N_s \to 0} s(\tau) = \infty, \ \lim_{N_e \to 0} e(\tau) = \infty, \ \lim_{N_s \to \infty} s(\tau) = 0, \ \lim_{N_e \to \infty} e(\tau) = 0.$$

Proof By substituting $\nu_e = 1$ and $\nu_s = 1$ (and, hence, $\varphi_e = \frac{1-\nu_e}{\nu_e} = 0$ and $\varphi_s = \frac{1-\nu_s}{\nu_s} = 0$) into equation (C.6) and (C.7), we have

$$\begin{aligned} s(\tau) &= N_s^{-1} (\nu_s A_s)^{\sigma_s} \alpha_s(\tau)^{\sigma_s} \alpha_h(\tau)^{-\frac{\sigma_e}{1-\sigma_e}} b(\hat{h}(\tau), \tau) \mu(\hat{h}(\tau)) \hat{h}'(\tau) \\ &\times \left[\left(p(\tau)^{1-\sigma_s} - \alpha_s(\tau)^{\sigma_s} (\nu_s A_s)^{\sigma_s - 1} \right)^{\frac{1-\sigma_e}{1-\sigma_s}} - \alpha_e(\tau)^{\sigma_e} (\nu_e A_e)^{\sigma_e - 1} \right]^{\frac{\sigma_e}{1-\sigma_e}} \\ &\times \left(p(\tau)^{1-\sigma_s} - \alpha_s(\tau)^{\sigma_s} (\nu_s A_s)^{\sigma_s - 1} \right)^{\frac{\sigma_s - \sigma_e}{1-\sigma_s}}, \end{aligned} \tag{C.10}$$

and

$$\begin{aligned} e(\tau) &= N_e^{-1} (\nu_e A_e)^{\sigma_e} \alpha_e(\tau)^{\sigma_e} \alpha_h(\tau)^{-\frac{\sigma_e}{1-\sigma_e}} b(\hat{h}(\tau), \tau) \mu(\hat{h}(\tau)) \hat{h}'(\tau) \\ &\times \left[\left(p(\tau)^{1-\sigma_s} - \alpha_s(\tau)^{\sigma_s} (\nu_s A_s)^{\sigma_s - 1} \right)^{\frac{1-\sigma_e}{\cdot}} 1 - \sigma_s - \alpha_e(\tau)^{\sigma_e} (\nu_e A_e)^{1-\sigma_e} \right]^{\frac{\sigma_e}{1-\sigma_e}} \end{aligned} \tag{C.11}$$

The result is straightforward from equations (C.10) and (C.11). ∎

Since π_e and π_s are proportional to the integration of $s(\tau)$ and $e(\tau)$, lemma 4 and 5 imply the existence of a unique steady state under some large enough ν_e and ν_s, fixing the static equilibrium.

Note that both $\hat{h}$ and $\mu(h)dh$ are bounded above by assumption and boundary conditions, and $p(\tau)$ is also bounded as $\int_\tau \gamma(\tau) p(\tau)^{1-\epsilon} d\tau = 1$. Hence, the existence follows when π_e and π_s are continuous in N_e and N_s, even when considering changes in static equilibrium. Recall that $p(\tau)$ and $\hat{h}(\tau)$ can be obtained from the system of differential equations (9) to (11). Since all functions in equation (9) to (11) are differentiable, π_e and π_s are also continuous in N_e and N_s, and the desired result follows.

Intuitively, large ν_e and ν_s mean small returns to introducing additional variety, thus meaning a decreasing rate of return. To see this intuition more clearly, recall that the task production function is given *by*

$$T(\tau) = \left[\left\{ \alpha_h(\tau) \left(b(\hat{h}(\tau),\tau)\mu(\hat{h}(\tau))\hat{h}'(\tau) \right)^{\frac{\sigma_e-1}{\sigma_e}} + \alpha_e(\tau) N_e^{\frac{\sigma_e-1}{\sigma_e \nu_e}} e(\tau)^{\frac{\sigma_e-1}{\sigma_e}} \right\}^{\frac{\sigma_e(\sigma_s-1)}{(\sigma_e-1)\sigma_s}} \right.$$
$$\left. + \alpha_s(\tau) N_s^{\frac{\sigma_s-1}{\sigma_s \nu_s}} s(\tau)^{\frac{\sigma_s-1}{\sigma_s}} \right]^{\frac{\sigma_s}{\sigma_s-1}}, \tag{C.12}$$

as $s(k,\tau) = s(\tau)$ and $e(k,\tau) = e(\tau)$ in equilibrium. The production is homogeneous of degree one in labor, N_e and N_s when $\nu_e \to 1$ and $\nu_s \to 1$. Since labor is a fixed component, the production features strict concavity along N_e and N_s, meaning decreasing returns to scale in terms of total varieties.

The second part of lemma (3) is when $\sigma_e = \sigma_s = 1$. In this case,

$$p(\tau)T(\tau) = \frac{\omega(\tau) b(\hat{h}(\tau),\tau)\mu(\hat{h}(\tau))\hat{h}'(\tau)}{\alpha_h(\tau)}, \tag{C.13}$$
$$s(\tau) = \frac{\nu_s A_s \alpha_s(\tau) p(\tau) T(\tau)}{N_s}, \tag{C.14}$$
$$e(\tau) = \frac{\nu_e A_e \alpha_e(\tau) p(\tau) T(\tau)}{N_e}. \tag{C.15}$$

Combining the FOCs, $T(\tau)$ satisfies

$$p(\tau)T(\tau) = p(\tau)^{\frac{1}{\alpha_h(\tau)}} \kappa(\tau) N_s^{\Psi_{es}(\tau)} \left(\frac{N_e}{N_s} \right)^{\Psi_e(\tau)} B(\tau), \tag{C.16}$$

where $\kappa(\tau) := (\alpha_s(\tau)\nu_s A_s)^{\frac{\alpha_s(\tau)}{\alpha_h(\tau)}} (\alpha_e(\tau)\nu_e A_e)^{\frac{\alpha_e(\tau)}{\alpha_h(\tau)}}$, $\Psi_{es}(\tau) := \frac{1-\nu_s}{\nu_s}\frac{\alpha_s(\tau)}{\alpha_h(\tau)} + \frac{1-\nu_e}{\nu_e}\frac{\alpha_e(\tau)}{\alpha_h(\tau)}$, $\Psi_e(\tau) := \frac{1-\nu_e}{\nu_e}\frac{\alpha_e(\tau)}{\alpha_h(\tau)}$, and $B(\tau) := b(\hat{h}(\tau),\tau)\mu(\hat{h}(\tau))\hat{h}'(\tau)$ are introduced to simplify the notation.

From equation (C.14) and (C.15), it is apparent that $s(\tau)$ and $e(\tau)$ are decreasing in N_s and N_e, respectively, when $\Psi_{es}(\tau) < 1$, which is a condition given in lemma 3. ∎

Proof of proposition 1 (job polarization) *Substituting $p(\tau)$ out from equation (9) to (11), we have*

$$\hat{h}'(\tau) = \frac{\gamma(\tau)\alpha_h(\tau)^{1-\alpha_h(\tau)(1-\epsilon)} Y}{b(\hat{h}(\tau),\tau)\mu(\hat{h}(\tau))\omega(\tau)^{1-\alpha_h(\tau)(1-\epsilon)}} \times$$
$$\left[\left(\alpha_s(\tau)\nu_s A_s N_s^{(1-\nu_s)/\nu_s} \right)^{\alpha_s(\tau)} \left(\alpha_e(\tau)\nu_e A_e N_e^{(1-\nu_e)/\nu_e} \right)^{\alpha_e(\tau)} \right]^{\epsilon-1} \tag{C.17}$$
$$\frac{d\log\omega(\tau)}{d\tau} = -\frac{\partial \log b(\hat{h}(\tau),\tau)}{\partial \tau} \tag{C.18}$$

First, we show that $\hat{h}_1$ and $\hat{h}_2$ must cross at least once. Suppose there is no crossing. Since $\hat{h}_1(0) =$

$\hat{h}_2(0)$ *and* $\hat{h}_1(\bar{\tau}) = \hat{h}_2(\bar{\tau})$*, we have*

$$\left(\frac{\omega_1(0)}{\omega_2(0)}\right)^{1-\alpha_h(0)(1-\epsilon)} = \frac{\hat{h}_2'(0)}{\hat{h}_1'(0)}\left(\frac{A_{e2}}{A_{e1}}\right)^{(1-\epsilon)\alpha_e(0)}, \tag{C.19}$$

$$\left(\frac{\omega_1(\bar{\tau})}{\omega_2(\bar{\tau})}\right)^{1-\alpha_h(\bar{\tau})(1-\epsilon)} = \frac{\hat{h}_2'(\bar{\tau})}{\hat{h}_1'(\bar{\tau})}\left(\frac{A_{e2}}{A_{e1}}\right)^{(1-\epsilon)\alpha_e(\bar{\tau})}, \tag{C.20}$$

from equation (C.17). Combining

$$\left(\frac{\omega_1(\bar{\tau})/\omega_1(0)}{\omega_2(\bar{\tau})/\omega_2(0)}\right)^{1-\alpha_h(0)(1-\epsilon)}\left(\frac{\omega_1(\bar{\tau})}{\omega_2(\bar{\tau})}\right)^{(\alpha_h(0)-\alpha_h(\bar{\tau}))(1-\epsilon)} = \frac{\hat{h}_2'(\bar{\tau})/\hat{h}_2'(0)}{\hat{h}_1'(\bar{\tau})/\hat{h}1'(0)} \tag{C.21}$$

Since $\hat{h}(\tau)$ *is strictly monotone and continuous, with no crossing on entire* $(0,\bar{\tau})$*, we have to have either (i)* $\hat{h}_2'(\bar{\tau})/\hat{h}_2'(0) < \hat{h}_1'(\bar{\tau})/\hat{h}_1'(0)$ *and* $\hat{h}_1(\tau) < \hat{h}_2(\tau)$ *for* $\tau \in (0,\bar{\tau})$*, or (ii)* $\hat{h}_2'(\bar{\tau})/\hat{h}_2'(0) > \hat{h}_1'(\bar{\tau})/\hat{h}_1'(0)$ *and* $\hat{h}_1(\tau) > \hat{h}_2(\tau)$ *for* $\tau \in (0,\bar{\tau})$*. However, from equation (C.18) and log supermodularity of* $b(h,\tau)$*, we have* $\omega_1(\bar{\tau})/\omega_1(0) > \omega_2(\bar{\tau})/\omega_2(0)$ *with* $\hat{h}_1(\tau) < \hat{h}_2(\tau)$*. With small enough* $\alpha_s(\bar{\tau})$*,* $(\omega_1(\bar{\tau})/\omega_2(\bar{\tau}))^{(\alpha_h(0)-\alpha_h(\bar{\tau}))(1-\epsilon)}$ *approaches one, and hence, equation (C.21) contradicts log supermodularity of* $b(h,\tau)$*.*

Second, we show that when $\hat{h}_1(\tau)$ *and* $\hat{h}_2(\tau)$ *cross at any three points* $\tau_a < \tau_b < \tau_c$*, we have* $\hat{h}_1'(\tau_a)/\hat{h}_1'(\tau_b) < \hat{h}_2'(\tau_a)/\hat{h}_2'(\tau_b)$ *with* $\hat{h}_2(\tau) > \hat{h}_1(\tau)$ *for* $\tau \in (\tau_a,\tau_b)$ *and* $\hat{h}_1'(\tau_c)/\hat{h}_1'(\tau_b) < \hat{h}_2'(\tau_c)/\hat{h}_2'(\tau_b)$ *with* $\hat{h}_1(\tau) > \hat{h}_2(\tau)$ *for* $\tau \in (\tau_b,\tau_c)$*.*

From equilibrium condition (C.17),

$$\left(\frac{\omega_1(\tau_b)/\omega_1(\tau_a)}{\omega_2(\tau_b)/\omega_2(\tau_a)}\right)^{1-\alpha_h(\tau_a)(1-\epsilon)}\left(\frac{\omega_1(\tau_b)}{\omega_2(\tau_b)}\right)^{(\alpha_h(\tau_a)-\alpha_h(\tau_b))(1-\epsilon)} = \frac{\hat{h}_2'(\tau_b)/\hat{h}_2'(\tau_a)}{\hat{h}_1'(\tau_b)/\hat{h}_1'(\tau_a)}\left(\frac{A_{e2}}{A_{e1}}\right)^{(1-\epsilon)(\alpha_e(\tau_b)-\alpha_e(\tau_a))} \tag{C.22}$$

$$\left(\frac{\omega_1(\tau_c)/\omega_1(\tau_b)}{\omega_2(\tau_c)/\omega_2(\tau_b)}\right)^{1-\alpha_h(\tau_c)(1-\epsilon)}\left(\frac{\omega_1(\tau_b)}{\omega_2(\tau_b)}\right)^{(\alpha_h(\tau_b)-\alpha_h(\tau_c))(1-\epsilon)} = \frac{\hat{h}_2'(\tau_c)/\hat{h}_2'(\tau_b)}{\hat{h}_1'(\tau_c)/\hat{h}_1'(\tau_b)}\left(\frac{A_{e2}}{A_{e1}}\right)^{(1-\epsilon)(\alpha_e(\tau_c)-\alpha_e(\tau_b))} \tag{C.23}$$

With small enough $\alpha_h'(\tau)$*, these equations are approximated to*

$$\left(\frac{\omega_1(\tau_b)/\omega_1(\tau_a)}{\omega_2(\tau_b)/\omega_2(\tau_a)}\right)^{1-\alpha_h(\tau_a)(1-\epsilon)} \approx \frac{\hat{h}_2'(\tau_b)/\hat{h}_2'(\tau_a)}{\hat{h}_1'(\tau_b)/\hat{h}_1'(\tau_a)}\left(\frac{A_{e2}}{A_{e1}}\right)^{(1-\epsilon)(\alpha_e(\tau_b)-\alpha_e(\tau_a))} \tag{C.24}$$

$$\left(\frac{\omega_1(\tau_c)/\omega_1(\tau_b)}{\omega_2(\tau_c)/\omega_2(\tau_b)}\right)^{1-\alpha_h(\tau_c)(1-\epsilon)} \approx \frac{\hat{h}_2'(\tau_c)/\hat{h}_2'(\tau_b)}{\hat{h}_1'(\tau_c)/\hat{h}_1'(\tau_b)}\left(\frac{A_{e2}}{A_{e1}}\right)^{(1-\epsilon)(\alpha_e(\tau_c)-\alpha_e(\tau_b))} \tag{C.25}$$

The only possibility that this can hold at the same time is when $\alpha_e(\tau_b) > \alpha_e(\tau_a)$ *and* $\alpha_e(\tau_b) > \alpha_e(\tau_c)$ *so that the signs of exponent term with respect to* (A_{e2}/A_{e1}) *are different. Recall that* $\omega_1(\tau_b)/\omega_1(\tau_a) < \omega_2(\tau_b)/\omega_2(\tau_a)$ *implies* $\hat{h}_2'(\tau_b)/\hat{h}_2'(\tau_a) > \hat{h}_1'(\tau_b)/\hat{h}_1'(\tau_a)$ *from equilibrium condition (C.18) and log su-*

permodularity of $b(h,\tau)$. Since $q_{e1} > q_{e2}$, $\alpha_e(\tau_b) > \alpha_e(\tau_a)$, and $\alpha_e(\tau_b) > \alpha_e(\tau_c)$, we must have $\omega_1(\tau_b)/\omega_1(\tau_a) > \omega_2(\tau_b)/\omega_2(\tau_a)$ and $\omega_1(\tau_c)/\omega_1(\tau_b) < \omega_2(\tau_c)/\omega_2(\tau_b)$, which implies $\hat{h}_1(\tau) < \hat{h}_2(\tau)$ for $\tau \in (\tau_a, \tau_b)$ and $\hat{h}_1(\tau) > \hat{h}_2(\tau)$ for $\tau \in (\tau_b, \tau_c)$.

The proof in the first part rules out any even number of crossings and no crossing. The second part implies they have to cross only a single time on $\tau \in (0, \bar{\tau})$ because they already meet at 0 and $\bar{\tau}$. Then, the result follows from the second part of the proof. ∎

Proof of proposition 2 (the rise of software) *We first show that the production share of the middle-skill task (task 1) falls and that of a high-skill task (task 2) rises in response to the decline in the price of equipment in a discretized model as well. To be specific, we prove the following lemma first.*

Lemma 6 *Fix N_e and N_s. Consider a decline in the price of equipment; $d\log A_e > 0$ and suppose $\epsilon < 1$ and assumption 2, 5, and 6. Then, we have $d\log p_1 < 0$ and $d\log p_2 > 0$.*

Proof From the equilibrium conditions (B.1) to (B.3),

$$\sum_{j=0}^{2} \gamma_j p_j^{1-\epsilon} = 1$$

$$p_j = \left(\frac{\omega_j}{\alpha_{h,j}}\right)^{\alpha_{h,j}} \left(\frac{1}{\nu_e A_e \alpha_{e,j}}\right)^{\alpha_{e,j}} \left(\frac{1}{\nu_s A_s \alpha_{s,j}}\right)^{\alpha_{s,j}} N_e^{-\varphi_e \alpha_{e,j}} N_s^{-\varphi_s \alpha_{s,j}}, \text{ for } j = 0,1,2$$

$$w_{j-1} b(\hat{h}_j, j-1) = w_j b(\hat{h}_j, j), \text{ for } j = 1,2$$

$$\frac{\omega_{j-1}\int_{\hat{h}_{j-1}}^{\hat{h}_j} b(h,j-1)\mu(h)dh}{\omega_j \int_{\hat{h}_j}^{\hat{h}_{j+1}} b(h,j)\mu(h)dh} = \frac{\alpha_{h,j-1}\gamma_{j-1}}{\alpha_{h,j}\gamma_j}\left(\frac{p_{j-1}}{p_j}\right)^{1-\epsilon}, \text{ for } j = 1,2,$$

with $\sigma_s = \sigma_e = 1$.

Let $\Delta x = d\log(x)$. Then, by differentiating the above and using assumption 5,

$$\Delta p_j = \alpha_{h,j}\Delta\omega_j - \alpha_{e,j}\Delta A_e \tag{C.26}$$

$$\Delta\omega_{j-1} = \Delta\omega_j + \Delta b(\hat{h}_j, j) - \Delta b(\hat{h}_j, j-1) \tag{C.27}$$

$$\Delta\omega_{j-1} - \Delta\omega_j = (1-\epsilon)(\Delta p_{j-1} - \Delta p_j) \tag{C.28}$$

$$\sum_{j=0}^{2} \gamma_j p_j^{1-\epsilon}\Delta p_j = 0 \tag{C.29}$$

Eliminating ω_j's,

$$\left(\frac{1}{\alpha_{h,0}} - (1-\epsilon)\right)\Delta p_0 = \left(\frac{1}{\alpha_{h,1}} - (1-\epsilon)\right)\Delta p_1 + \left(\frac{\alpha_{e,1}}{\alpha_{h,1}} - \frac{\alpha_{e,0}}{\alpha_{h,0}}\right)\Delta A_e \tag{C.30}$$

$$\left(\frac{1}{\alpha_{h,2}} - (1-\epsilon)\right)\Delta p_2 = \left(\frac{1}{\alpha_{h,1}} - (1-\epsilon)\right)\Delta p_1 + \left(\frac{\alpha_{e,1}}{\alpha_{h,1}} - \frac{\alpha_{e,2}}{\alpha_{h,2}}\right)\Delta A_e \tag{C.31}$$

Since $1/\alpha_{h,j} > (1-\epsilon)$ for all j's and $\alpha_{e,1}/\alpha_{h,1} > \alpha_{e,j}/\alpha_{h,j}$ for $j = 0, 2$, it is easy to check that $\Delta p_1 < 0$ by substituting equation (C.30) and (C.31) into equation (C.29).

Substituting equation (C.30) and (C.31) into equation (C.29), we also have

$$\begin{aligned}
&\left[\gamma_0 p_0^{1-\epsilon}\left(\frac{\frac{1}{\alpha_{h,2}}-(1-\epsilon)}{\frac{1}{\alpha_{h,0}}-(1-\epsilon)}\right)+\gamma_2 p_2^{1-\epsilon}+\gamma_1 p_1^{1-\epsilon}\left(\frac{\frac{1}{\alpha_{h,2}}-(1-\epsilon)}{\frac{1}{\alpha_{h,1}}-(1-\epsilon)}\right)\right]\Delta p_2 \\
&+\gamma_0 p_0^{1-\epsilon}\left[\frac{\left(\frac{\alpha_{e,1}}{\alpha_{h,1}}-\frac{\alpha_{e,0}}{\alpha_{h,0}}\right)-\left(\frac{\alpha_{e,1}}{\alpha_{h,1}}-\frac{\alpha_{e,2}}{\alpha_{h,2}}\right)}{\frac{1}{\alpha_{h,0}}-(1-\epsilon)}\right]\Delta A_e \\
&-\gamma_1 p_1^{1-\epsilon}\frac{\frac{\alpha_{e,1}}{\alpha_{h,1}}-\frac{\alpha_{e,2}}{\alpha_{h,2}}}{\frac{1}{\alpha_{h,1}}-(1-\epsilon)}\Delta A_e = 0
\end{aligned} \tag{C.32}$$

By assumption 6 and $\epsilon < 1$, we have

$$\begin{aligned}
&\left[\gamma_0 p_0^{1-\epsilon}\left(\frac{\frac{1}{\alpha_{h,2}}-(1-\epsilon)}{\frac{1}{\alpha_{h,0}}-(1-\epsilon)}\right)+\gamma_2 p_2^{1-\epsilon}+\gamma_1 p_1^{1-\epsilon}\left(\frac{\frac{1}{\alpha_{h,2}}-(1-\epsilon)}{\frac{1}{\alpha_{h,1}}-(1-\epsilon)}\right)\right] > 0, \\
&\gamma_0 p_0^{1-\epsilon}\left[\frac{\left(\frac{\alpha_{e,1}}{\alpha_{h,1}}-\frac{\alpha_{e,0}}{\alpha_{h,0}}\right)-\left(\frac{\alpha_{e,1}}{\alpha_{h,1}}-\frac{\alpha_{e,2}}{\alpha_{h,2}}\right)}{\frac{1}{\alpha_{h,0}}-(1-\epsilon)}\right] = 0, \\
&\gamma_1 p_1^{1-\epsilon}\frac{\frac{\alpha_{e,1}}{\alpha_{h,1}}-\frac{\alpha_{e,2}}{\alpha_{h,2}}}{\frac{1}{\alpha_{h,1}}-(1-\epsilon)} > 0,
\end{aligned}$$

implying $\Delta p_2 > 0$ from equation (C.32). ∎

Now, we show that lemma 6 implies a relative increase in software variety in the new steady state. Note that the profits from providing software and equipment variety are given by

$$\pi_s = \sum_j \frac{1-\nu}{\nu A_s} s_j \quad \textit{and} \quad \pi_e = \sum_j \frac{1-\nu}{\nu A_e} e_j.$$

From the FOC and using (C.4) ($p_e = 1/(\nu A_e)$ and $p_s = 1/(\nu A_s)$), demand for equipment and software for each task are $e_j = \nu_e A_e \alpha_{e,j} p_j T_j / N_e$ and $s_j = \nu_s A_s \alpha_{s,j} p_j T_j / N_s$.

From lemma 3, we know $\pi_e/\eta = \pi_s/\eta = \rho$ in any steady state equilibrium, and hence,

$$d\pi_e = (1-\nu)\Bigg[(1-\epsilon)(\alpha_{e,0}p_0^{-\epsilon}dp_0 + \alpha_{e,1}p_1^{-\epsilon}dp_1 + \alpha_{e,2}p_2^{-\epsilon}dp_2)Y$$

$$+(\sum_j \alpha_{e,j}p_j^{1-\epsilon})dY - \frac{1}{N_e}\sum_j \alpha_{e,j}p_j^{1-\epsilon}YdN_e\Bigg] = 0$$

$$d\pi_s = (1-\nu)\Bigg[(1-\epsilon)(\alpha_{s,0}p_0^{-\epsilon}dp_0 + \alpha_{s,1}p_1^{-\epsilon}dp_1 + \alpha_{s,2}p_2^{-\epsilon}dp_2)Y$$

$$+(\sum_j \alpha_{s,j}p_j^{1-\epsilon})dY - \frac{1}{N_s}\sum_j \alpha_{e,j}p_j^{1-\epsilon}YdN_s\Bigg] = 0$$

In combination,

$$\begin{aligned}
&(1-\epsilon)[(\alpha_{e,1}-\alpha_{s,1})p_1^{-\epsilon}dp_1 + (\alpha_{e,2}-\alpha_{s,2})p_2^{-\epsilon}dp_2] \\
&= \sum_j \alpha_{e,j}p_j^{1-\epsilon}\left[\frac{dN_e}{N_e} - \frac{dY}{Y}\right] - \sum_j \alpha_{s,j}p_j^{1-\epsilon}\left[\frac{dN_s}{N_s} - \frac{dY}{Y}\right] \\
&= \sum_j \alpha_{s,j}p_j^{1-\epsilon}\left[\frac{dN_e - dN_s}{N_s} - \left(1 - \frac{N_e}{N_s}\right)\frac{dY}{Y}\right] < 0,
\end{aligned}$$

where the last equality is from the no arbitrage condition (16) ($\frac{N_s}{N_e} = \frac{\sum_j \alpha_{s,j}\gamma_j p_j^{1-\epsilon}}{\sum_j \alpha_{e,j}\gamma_j p_j^{1-\epsilon}}$), and the inequality is from lemma 6 and assumption 6.

Hence, we have

$$dN_s > dN_e + (N_e - N_s)\frac{dY}{Y}.$$

Since a decrease in the price of equipment raises the level of production, we have $dY/Y > 0$. Hence, with the condition given in this proposition ($N_e \geq N_s$), $(N_e - N_s)dY/Y \geq 0$, and therefore, $dN_s > dN_e$. Finally, since $N_e \geq N_s$, we have

$$dN_s/N_s > dN_e/N_e,$$

as shown. ∎

Proof of proposition 3 (skill demand reversal) *Suppose they cross at least once, which means that we have at least three points $\tau_a < \tau_b < \tau_c$ such that $\hat{h}_1(\tau_a) = \hat{h}_2(\tau_a)$, $\hat{h}_1(\tau_b) = \hat{h}_2(\tau_b)$, and $\hat{h}_1(\tau_c) = \hat{h}_2(\tau_c)$.*

Then, we have

$$\left(\frac{\omega_1(\tau_b)/\omega_1(\tau_a)}{\omega_2(\tau_b)/\omega_2(\tau_a)}\right)^{1-\alpha_h(\tau_a)(1-\epsilon)}\left(\frac{\omega_1(\tau_b)}{\omega_2(\tau_b)}\right)^{(\alpha_h(\tau_a)-\alpha_h(\tau_b))(1-\epsilon)}=\frac{\hat{h}_2'(\tau_b)/\hat{h}_2'(\tau_a)}{\hat{h}_1'(\tau_b)/\hat{h}_1'(\tau_a)}\left(\frac{N_{s2}}{N_{s1}}\right)^{\varphi_s(1-\epsilon)(\alpha_s(\tau_b)-\alpha_s(\tau_a))} \tag{C.33}$$

$$\left(\frac{\omega_1(\tau_c)/\omega_1(\tau_b)}{\omega_2(\tau_c)/\omega_2(\tau_b)}\right)^{1-\alpha_h(\tau_c)(1-\epsilon)}\left(\frac{\omega_1(\tau_b)}{\omega_2(\tau_b)}\right)^{(\alpha_h(\tau_b)-\alpha_h(\tau_c))(1-\epsilon)}=\frac{\hat{h}_2'(\tau_c)/\hat{h}_2'(\tau_b)}{\hat{h}_1'(\tau_c)/\hat{h}_1'(\tau_b)}\left(\frac{N_{s2}}{N_{s1}}\right)^{\varphi_s(1-\epsilon)(\alpha_s(\tau_c)-\alpha_s(\tau_b))} \tag{C.34}$$

where $\varphi_s \equiv (1-\nu_s)/\nu_s$.

With a small enough $\alpha_h'(\tau)$, *the above equations can be approximated to*

$$\left(\frac{\omega_1(\tau_b)/\omega_1(\tau_a)}{\omega_2(\tau_b)/\omega_2(\tau_a)}\right)^{1-\alpha_h(\tau_a)(1-\epsilon)}\frac{\hat{h}_1'(\tau_b)/\hat{h}_1'(\tau_a)}{\hat{h}_2'(\tau_b)/\hat{h}_2'(\tau_a)}\approx\left(\frac{N_{s2}}{N_{s1}}\right)^{\varphi_s(1-\epsilon)(\alpha_s(\tau_b)-\alpha_s(\tau_a))} \tag{C.35}$$

$$\left(\frac{\omega_1(\tau_c)/\omega_1(\tau_b)}{\omega_2(\tau_c)/\omega_2(\tau_b)}\right)^{1-\alpha_h(\tau_c)(1-\epsilon)}\frac{\hat{h}_1'(\tau_c)/\hat{h}_1'(\tau_b)}{\hat{h}_2'(\tau_c)/\hat{h}_2'(\tau_b)}\approx\left(\frac{N_{s2}}{N_{s1}}\right)^{\varphi_s(1-\epsilon)(\alpha_s(\tau_c)-\alpha_s(\tau_b))} \tag{C.36}$$

Again, since the matching function is continuous and monotone and $b(h,\tau)$ *is log supermodular, the signs of the log of LHS in both equation (C.35) and equation (C.36) should be different. However, since* $\alpha_s(\tau)$ *is strictly increasing, signs of the log of RHS in equation (C.35) and equation (C.36) are the same, which is contradictory.*

Finally, to show $\hat{h}_2(\tau) < \hat{h}_1(\tau)$ *for* $\tau \in (0,\bar{\tau})$, *recall that equilibrium condition (C.17) implies*

$$\left(\frac{\omega_1(\bar{\tau})/\omega_1(0)}{\omega_2(\bar{\tau})/\omega_2(0)}\right)^{1-\alpha_h(\bar{\tau})(1-\epsilon)}\frac{\hat{h}_1'(\bar{\tau})/\hat{h}_1'(0)}{\hat{h}_2'(\bar{\tau})/\hat{h}_2'(0)}=\left(\left(\frac{N_{s2}}{N_{s1}}\right)^{\varphi_s}\frac{\omega_2(0)}{\omega_1(0)}\right)^{(1-\epsilon)(\alpha_s(\bar{\tau})-\alpha_s(0))} \tag{C.37}$$

Since $(1-\epsilon)(\alpha_s(\bar{\tau})-\alpha_s(0)) > 0$ *and* $N_{s2} > N_{s1}$, *we must have* $\omega_1(\bar{\tau})/\omega_1(0) > \omega_2(\bar{\tau})/\omega_2(0)$, *which implies* $\hat{h}_2(\tau) > \hat{h}_1(\tau)$. ∎

D Calibration Procedure

This section describes the detailed calibration procedure. We normalize exogenous variables M_j's, A_e, and A_s to one in 1980.

1. We begin with $\hat{h}_j$, which correspond to the employment share of occupation j in 1980 and fix ϵ, σ_s and σ_e arbitrarily.

2. By indifference between tasks at the threshold level of skills, we have

$$\frac{w_j}{w_{j-1}}=\frac{\hat{h}_j-\chi_{j-1}}{\hat{h}_j-\chi_j},$$

and thus, $w_j = w_0 \prod_{k=1}^{j} (\hat{h}_k - \chi_{k-1})/(\hat{h}_k - \chi_k)$. Therefore, the payroll share of occupation j is given by

$$\frac{\prod_{k=1}^{j} (\hat{h}_k - \chi_{k-1})/(\hat{h}_k - \chi_k) \int_{\hat{h}_{j-1}}^{\hat{h}_j} (h - \chi_j) h^{-a-1} dh}{\sum_j \prod_{k=1}^{j} (\hat{h}_k - \chi_{k-1})/(\hat{h}_k - \chi_k) \int_{\hat{h}_{j-1}}^{\hat{h}_j} (h - \chi_j) h^{-a-1} dh}.$$

We set 8 parameters χ_j's and 1 parameter a to minimize the distance between the payroll share in the data and the model for 9 occupations.

3. Guess $\alpha_{j,e}$ and $\alpha_{j,s}$. We find γ_j's that match with $\hat{h}_j$'s in equilibrium.

4. We iterate over $\alpha_{j,e}$ and $\alpha_{j,s}$ until the aggregate labor share E_j and S_j in the model match with the aggregate labor share, equipment and software investment by occupation in the data.

5. We solve for M_j's for routine occupations ($j = 2, 3, 4, 5, 7, 8$) to match the employment share of routine occupations in the data. Note that we already have different values of A_e and A_s for each period obtained from the data.

6. Iterate over σ_s and σ_e so that the labor share with and without the software matches with the trend-implied level in 2010.

7. Iterate over ϵ in order to minimize an average distance between changes in the payroll share by occupation in the model and data.

The procedure gives all the parameters that need to be calibrated. For ν_e and ν_s, we use the estimated value as described in section 6.