Missing Risk Sharing from International Transmission through Product Quality and Variety*

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Abstract

This paper explores the role played by product quality and variety in international consumption risk sharing. Turnover in product quality and variety can cause a wealth effect. A reasonable Backus-Smith correlation is driven by the Harrod-Balassa-Samuelson mechanism based on heterogeneous firms. Using panel data, we test the prediction of the theoretical model and find a supportive evidence for a resolution to the Backus-Smith puzzle. The extent of "missing risk sharing" due to unobservable fluctuations in quality and the number of varieties is high in the data.

Keywords: *exchange rate, consumption-real exchange rate anomaly, product quality, firm heterogeneity*

JEL classification: *F12, F41, F43*

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1 Introduction

To what extent do households living in different countries insure against each other's consumption risk? The degree of consumption risk sharing depends on the availability of internationally held financial assets. It can be complemented by fluctuations in relative prices of exchanged goods (Cole and Obstfeld, 1991). Simultaneously, the quality of traded products and the number of traded varieties change over time, which could provide a rich international transmission mechanism.

Based on Ghironi and Melitz (2005), this paper provides a two-country DSGE model that captures changes in both the number of product varieties and product quality produced by heterogeneous firms. In particular, we explore the implication of endogenous fluctuations in product quality and variety on international consumption risk sharing, namely, the Backus-Smith puzzle (Backus and Smith, 1993, Kollmann, 1995). We use panel data on advanced economies to test the prediction of the model and show that product quality and the number of product varieties play a key role in resolving the puzzle.

If financial markets that allow to hedge any consumption risk ex ante exist, we should expect to see a strong comovement of consumption growth across countries despite idiosyncratic income shock. By explicitly considering the difference in the price level of consumption goods across countries, under such complete asset markets, a country's consumption increases exactly when the real exchange rate *depreciates* for that country, providing a positive correlation between relative consumption and real exchange rate fluctuations. However, the correlation between relative consumption and fluctuations in the real exchange rate is close to zero or even negative for major advanced economies, which is known as the consumption-real exchange rate anomaly or the Backus-Smith puzzle. Table 1 provides correlations between the growth rate of per capita consumption with respect to the sample average and the growth rate of the real effective exchange rate for 26 major economies. The table shows puzzling correlations that are close to zero or even negative. The mean correlation is -0.10.

How does international transmission arising from product quality and variety modify

Table 1: The Backus-Smith Correlation

Australia (AUS)	-0.26	France (FRA)	0.30	Netherlands (NLD)	-0.38
Austria (AUT)	-0.59	United Kingdom (GBR)	0.24	Norway (NOR)	0.10
Belgium (BEL)	-0.34	Greece (GRC)	-0.34	New Zealand (NZL)	0.21
Canada (CAN)	-0.15	Hong Kong (HKG)	0.14	Portugal (PRT)	-0.91
Switzerland (CHE)	-0.33	Ireland (IRL)	-0.66	Singapore (SGP)	-0.10
Germany (DEU)	-0.23	Italy (ITA)	-0.10	Sweden (SWE)	0.44
Denmark (DNK)	-0.40	Japan (JPN)	-0.04	Taiwan (TWN)	0.49
Spain (ESP)	-0.02	Korea (KOR)	0.53	United States (USA)	0.19
Finland (FIN)	0.15	Mexico (MEX)	-0.58	Mean	-0.10

Note: Data on per capita consumption and the real effective exchange rate come from Penn World Table 9.1 and the narrow indices of BIS, respectively, from 1984 to 2011.

the nature of international risk sharing? The reason why the theoretical model fails to reproduce such a negative Backus-Smith correlation can be attributed to the presence of a complete financial market (Obstfeld and Rogoff, 2000) from which a tight positive correlation between relative consumption and real exchange rate fluctuations emerges. In contrast, assuming a weak role of financial assets in hedging consumption risk, Corsetti et al. (2008) emphasize the "wealth effect" with which the real exchange rate *appreciates* at the same time the level of consumption increases in that country. In Corsetti et al. (2008), a relatively strong wealth effect together with a reasonable Backus-Smith correlation are obtained due to a lower value of elasticity of substitution between local and imported goods and/or a high persistence of productivity shock. Hamano (2013) shows that the wealth effect with which a reasonable Backus-Smith correlation arises can be obtained with the entry of new product varieties. Different from the preceding literature, in our model, the wealth effect is driven by a higher number of product varieties and/or a higher quality of products produced by heterogeneous firms. Thus, the mechanism in our paper hinges on a very sophisticated Harrod-Balassa-Samuelson effect based on heterogeneous firms, as documented in Ghironi and Melitz (2005).[1]

In our paper, it is analytically shown that the Backus-Smith correlation is structurally a *conditional* correlation on changes in product quality and the number of varieties. Importantly, we test this prediction of the theoretical model with actual data. Using a panel dataset of major economies that includes the number of exported and imported goods and export and import quality, we *condition* the Backus-Smith relation on fluctuations in product quality and the number of traded varieties. We find that the conditional Backus-Smith correlation tends to become positive, resolving the original puzzling correlation and indicating better risk sharing across countries. We also perform a robustness check relying on regression analysis and find supportive evidence.

Finally, fluctuations in product quality and variety very naturally break the tight link between relative consumption and the real exchange rate fluctuations, even under complete financial markets. Since statistical institutions only imperfectly capture turnover in product quality and the number of product varieties (Broda and Weinstain (2004, 2006)), the observable consumption risk sharing is based on noisy consumption and imperfectly measured fluctuations in the real exchange rate. Thus, our paper echoes the recent contribution by Aghion et al. (2017) that discusses the "missing growth" due to systematic measurement error, followed by a Schumpeterian creative destruction process. Our paper documents the extent of "missing risk sharing" in international business cycles.

The paper is organized as follows. In the next section, we present the model. In Section 3, we analytically investigate the nature of international risk sharing and transmission with a linearized system of equations. We next calibrate the model and document its quantitative implications. An empirical investigation is conducted in Section 5. In the last section, we conclude.

[1] In Benigno and Thoenissen (2008), with exogenously determined traded and non-traded sectors, the appreciation in the real exchange rate is driven by the well-known standard Harrod-Balassa-Samuelson effect and, thus, the wealth effect due to the presence of the non-traded sector.

2 The model

2.1 Household Preferences and Intratemporal Choices

The world consists of two countries, Home and Foreign. Foreign variables are denoted with an asterisk (*). Each country is populated by one unit mass of atomic households. We discuss the representative household in Home. A similar argument holds for households in Foreign. The Home representative household maximizes expected intertemporal utility, $E_t \sum_{s=t}^{\infty} \beta^{s-t} U_t$, where β $(0 < \beta < 1)$ is the exogenous discount factor. The utility at time t depends on consumption and the labor supply as follows

$$U_t = \frac{C_t^{1-\gamma}}{1-\gamma} - \chi \frac{L_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}},$$

In the above expression, γ (≥ 1) denotes risk aversion. χ (> 0) represents the degree of non-satisfaction from supplying labor L_t, and φ (≥ 0) denotes Frisch elasticity of the labor supply.

The basket of goods C_t is defined as

$$C_t = \left[C_{H,t}^{1-\frac{1}{\omega}} + C_{F,t}^{1-\frac{1}{\omega}} \right]^{\frac{1}{1-\frac{1}{\omega}}},$$

where α $(> 1/2)$ is home bias in consumption attached to the bundle of goods produced locally, $C_{H,t}$. ω (> 0) denotes the elasticity of substitution between local $(C_{H,t})$ and imported goods $(C_{F,t})$. $C_{H,t}$ and $C_{F,t}$ are defined over a continuum of goods Ω:

$$C_{H,t} = V_{H,t} \left(\int_{\zeta \in \Omega} q_D(\zeta)\, c_{D,t}(\zeta)^{1-\frac{1}{\sigma}}\, d\zeta \right)^{\frac{1}{1-\frac{1}{\sigma}}}, \quad C_{F,t} = V_{F,t}^{*} \left(\int_{\vartheta \in \Omega} q_X^{*}(\vartheta)\, c_{X,t}(\vartheta)^{1-\frac{1}{\sigma}}\, d\vartheta \right)^{\frac{v1}{1-\frac{1}{\sigma}}},$$

where $V_{H,t} \equiv N_{D,t}^{\psi - \frac{1}{\sigma-1}}$ and $V_{F,t}^{*} \equiv N_{X,t}^{*\psi - \frac{1}{\sigma-1}}$. $N_{D,t}$ and $N_{X,t}^{*}$ stand for the number of domestic and imported product varieties. ψ (≥ 0) represents the marginal utility that stems from one additional increase in the number of varieties in each basket (Benassy 1996). Specifically, the preference becomes Dixit and Stiglitz (1977) when $\psi = \frac{1}{\sigma-1}$. At any given time t, only a subset of goods $\Omega_t \in \Omega$ is available. $c_{D,t}(\zeta)$ and $c_{X,t}(\vartheta)$ represent the demand addressed for individual product variety ζ and ϑ, which are produced domestically and imported, respectively. $q_D(\zeta)$ and $q_X(\vartheta)$ denote the quality of these product varieties and act as a demand sifter. σ (>1) denotes the elasticity of substitution among varieties. We assume conventionally $\sigma \geq \omega$.

The optimal consumption for each domestic, imported basket and individual product variety is found to be

$$C_{H,t} = \left(\frac{P_{H,t}}{P_t}\right)^{-\omega} C_t, \qquad C_{F,t} = \left(\frac{P_{F,t}}{P_t}\right)^{-\omega} C_t.$$

$$c_{D,t}(\zeta) = \left(V_{H,t} q_D(\zeta)\right)^{\sigma-1} \left(\frac{p_{D,t}(\zeta)}{P_{H,t}}\right)^{-\sigma} C_{H,t}, \qquad c_{X,t}(\vartheta) = \left(V^*_{F,t} q^*_X(\vartheta)\right)^{\sigma-1} \left(\frac{p^*_{X,t}(\vartheta)}{P_{F,t}}\right)^{-\sigma} C_{F,t}.$$

In particular, $p^*_{X,t}(\vartheta)$ denotes the price of exported goods from Foreign.

Price indices that minimize expenditure on each consumption basket are given by

$$P_t = \left[P_{H,t}^{1-\omega} + P_{F,t}^{1-\omega}\right]^{\frac{1}{1-\omega}},$$

$$P_{H,t} = \frac{1}{V_{H,t}} \left(\int_{\zeta\in\Omega_t} \left(\frac{p_{D,t}(\zeta)}{q_D(\zeta)}\right)^{1-\sigma} d\zeta\right)^{\frac{1}{1-\sigma}}, \qquad P_{F.t} = \frac{1}{V^*_{F.t}} \left(\int_{\vartheta\in\Omega_t} \left(\frac{p^*_{X,t}(\vartheta)}{q^*_X(\vartheta)}\right)^{1-\sigma} d\vartheta\right)^{\frac{1}{1-\sigma}}.$$

Observe that the price indices defined on a welfare basis fluctuate with changes in the number of varieties and product quality. Finally, we choose the welfare-based consumer price index, P_t, as a numéraire in Home and define real prices as $\rho_{H,t} \equiv \frac{P_{H,t}}{P_t}$, $\rho_{F,t} \equiv \frac{P_{F,t}}{P_t}$, $\rho_{D,t}(\zeta) \equiv \frac{p_{D,t}(\zeta)}{P_t}$ and $\rho^*_{X,t}(\vartheta) \equiv \frac{p^*_{X,t}(\vartheta)}{P_t}$.

Similar expressions hold in Foreign. Crucially, the subset of goods available in Foreign during period t, $\Omega^*_t \in \Omega$, can be different from the subset of goods available in Home.

2.2 Production, Pricing and the Export Decision

In every period, there is a mass of $N_{E,t}$ entrants. Prior to entry, these new entrants are identical and face a sunk entry cost of f_E, which is defined as follows

$$f_E = Z_t l_{E,t},$$

where Z_t denotes the labor productivity level, which is specific to the firm setup and common for all firms. $l_{E,t}$ is the demand for labor in the firm setup. Upon entry, firms draw their productivity level z from a distribution $G(z)$ with support on $[z_{\min}, \infty)$. Since there are no fixed production costs, all firms produce unless they are hit by an exogenous depreciation shock, which occurs with probability $\delta \in (0,1)$ in every period. This exit-inducing shock is independent of the firm-specific productivity level and assumed to take place at the very end of the period. Thus, $G(z)$ also represents the productivity distribution of all producing firms.

Exporting requires an operational fixed cost f_X in every period. Specifically, f_X is defined as

$$f_X = Z_t l_{f_X,t},$$

where Z_t denotes the labor productivity level in production and is common for all firms. $l_{f_X,t}$ is the demand for labor required to produce f_X amount of fixed costs. Only a subset of firms with a productivity level z that is above the cutoff level $z_{X,t}$ exports by charging sufficiently lower quality-adjusted prices and earning positive profits despite the existence of a fixed export cost f_X. Thus, non-tradeness in the economy appears endogenously with changes in the cutoff productivity level.

The firm faces a residual demand curve with constant elasticity σ. The production scale is thus determined by the demand addressed to the firm. Profit maximization of the firm under flexible prices yields the following optimal real prices:

$$\rho_{D,t}(z) = \frac{\sigma}{\sigma - 1} mc_t(z),$$

where $mc_t(z)$ is the real marginal cost. We assume that producing higher-quality goods requires a higher marginal cost $mc_t(z)$ such that

$$mc_t(z) = \left(1 + \frac{q(z)^{\frac{1}{\phi}}}{z}\right) \frac{w_t}{Z_t z},$$

where ϕ $(0 \leq \phi < 1)$ is a parameter that determines the quality ladder and w_t denotes real wage. Provided a firm-specific productivity level z, the firm endogenously chooses its specific quality level $q(z)$. Specifically, the firm minimizes the quality-adjusted marginal cost $mc_t(z)/q(z)$. As a result, optimal quality is given by

$$q(z) = \left(\frac{\phi}{1-\phi} z\right)^{\phi}.$$

Provided $\phi > 0$, a highly productive firm produces high-quality goods. Observe that when there is no quality ladder ($\phi = 0$), all firms produce a similar quality of goods, irrespective of their specific productivity levels. In this case, the model becomes similar to that in Ghironi and Melitz (2005), which embodies no choice of quality.

Due to a fixed operational export cost f_X, the firm z may not export. If the firm exports, its export price is $\rho_{X,t}(z) = \tau_t \rho_{D,t}(z) Q_t^{-1}$. τ_t is the iceberg trade cost. Q_t is the real exchange rate defined as the price of foreign consumption goods in terms of home consumption goods as $Q_t \equiv P_t^*/P_t$. $\rho_{X,t}(z)$ is thus denominated in terms of the price of the consumption basket in the export market.

Total profits of the firm $d_t(z)$ can be decomposed into those from domestic sales $d_{D,t}(z)$ and from exporting sales $d_{X,t}(z)$: $d_t(z) = d_{D,t}(z) + d_{X,t}(z)$. Using the demand functions found previously, we can write profits in each market as

$$d_{D,t}(z) = \frac{1}{\sigma} N_{D,t}^{\psi(\omega-1)-1} \left(\frac{\rho_{D,t}(z)}{q(z)} \right)^{1-\omega} C_t,$$

$$d_{X,t}(z) = \frac{Q_t}{\sigma} N_{X,t}^{\psi(\omega-1)-1} \left(\frac{\rho_{X,t}(z)}{q(z)} \right)^{1-\omega} C_t^* - \frac{w_t f_X}{Z_t}, \text{ if firm } z \text{ exports,}$$

2.3 Firm Averages

Given a distribution $G(z)$, a mass of $N_{D,t}$ of domestically producing firms has a distribution of productivity levels over $[z_{\min}, \infty)$. Among these firms, there are $N_{X,t} = [1 - G(z_{X,t})] N_{D,t}$ exporters in Home. Following Melitz (2003), we define two average productivity levels, $\widetilde{z}_D$ for domestically producing firms and $\widetilde{z}_{X,t}$ for exporters, as follows:

$$\widetilde{z}_D \equiv \left[\int_{z_{\min}}^{\infty} z^{\sigma-1} dG(z) \right]^{\frac{1}{\sigma-1}}, \quad z \equiv \left[\frac{1}{1 - G(z_{X,t})} \int_{z_{X,t}}^{\infty} z^{\sigma-1} dG(z) \right]^{\frac{1}{\sigma-1}}.$$

These average productivity levels summarize all the information about the distribution of productivities. Provided these averages, we define the average real domestic and export prices as $\widetilde{\rho}_{D,t} \equiv \rho_{D,t}(\widetilde{z}_D)$ and $\widetilde{\rho}_{X,t} \equiv \rho_{X,t}(\widetilde{z}_{X,t})$, respectively. Similarly, the average domestic and average expor quality (AEQ) are provided by $\widetilde{q}_D \equiv q_D(\widetilde{z}_D)$ and $\widetilde{q}_{X,t} \equiv q_{X,t}(\widetilde{z}_{X,t})$. Also, we define average real profits from domestic sales and export sales as $\widetilde{d}_{D,t} \equiv d_{D,t}(\widetilde{z}_D)$ and $\widetilde{d}_{X,t} \equiv d_{X,t}(\widetilde{z}_{X,t})$. Finally, average real profits among all Home firms are given by $\widetilde{d}_t^s = \widetilde{d}_{D,t} + (N_{X,t}/N_{D,t}) \widetilde{d}_{X,t}$.

2.4 Firm Entry and Exit

We assume that entrants at time t start producing only at time $t+1$. These entrants discount the stream of their expected profits $\left\{\widetilde{d}_i^s\right\}_{i=t+1}^{\infty}$ using the household's discount factor adjusted by exogenous exit-inducing shock δ. Thus, their expected post-entry value is

$$\widetilde{v}_t^s = E_t \sum_{i=t+1}^{\infty} \beta^{i-t} \left(\frac{C_i}{C_t}\right)^{-\gamma} (1-\delta)^{s-t} \widetilde{d}_i^s$$

This firm value is the price of Home equities, that is, the value of mutual funds among heterogeneous firms with a Home origin. Entry occurs until this expected firm value is equalized with the entry cost, leading to the following free entry condition:

$$\widetilde{v}_t^s = \frac{w_t}{Z_t} f_E.$$

The timing of entry and production we discussed implies that the number of domestically producing firms evolves according to $N_{D,t} = (1-\delta)\left(N_{D,t-1} + N_{E,t-1}\right)$.

2.5 Parametrization of Productivity Draws

We assume the following Pareto distribution for $G(z)$:

$$G(z) = 1 - \left(\frac{z_{\min}}{z}\right)^k,$$

where $z_{\min}$ is the minimum productivity level and k $(> \sigma - 1)$ is a shape parameter. With this parametrization, we have

$$\widetilde{z}_D = z_{\min}\left[\frac{k}{k-(\sigma-1)}\right]^{\frac{1}{\sigma-1}}, \quad \widetilde{z}_{X,t} = z_{X,t}\left[\frac{k}{k-(\sigma-1)}\right]^{\frac{1}{\sigma-1}}$$

Additionally, the share of exporters in the total number of domestic firms is given by

$$\frac{N_{X,t}}{N_{D,t}} = z_{\min}^k \left(\widetilde{z}_{X,t}\right)^{-k} \left[\frac{k}{k-(\sigma-1)}\right]^{\frac{k}{\sigma-1}}.$$

In the end, there exists firms that earn zero profits from exports in such a way that $d_{X,t}\left(z_{X,t}\right) = 0$ with a cutoff productivity level, $z_{X,t}$. With the above Pareto distribution, this implies that

$$\widetilde{d}_{X,t} = \frac{w_t f_X}{Z_t} \frac{\sigma - 1}{k - (\sigma - 1)}.$$

2.6 Household Budget Constraint and Intertemporal Choices

There are two types of financial assets – equities and bonds – that are held only domestically. Here, we present the case of financial autarky. In a later section, we relax this assumption by allowing international borrowing and lending and present the model with state non-contingent bonds in the appendix. The quantitative implications will prove to be almost identical to those in the case of financial autarky.

The Home representative household finances the entry cost of new entrants, $N_{E,t}$, and all producing firms, $N_{D,t}$ in Home, at time t by purchasing a share of Home equities, $s_{h,t+1}$. Similarly, the Foreign representative household finances the entry cost of new entrants, $N^*_{E,t}$ and all producing firms, $N^*_{D,t}$, at time t by purchasing a share of Foreign equities, $s_{f,t+1}$. Gross returns of Home and Foreign equities between t and $t+1$ (in units of Home consumption) are given by

$$R^s_{h,t+1} \equiv (1-\delta)\,\frac{\widetilde{v}^s_{t+1}+\widetilde{d}^s_{t+1}}{\widetilde{v}^s_t},\quad R^s_{f,t+1} \equiv (1-\delta)\,\frac{\widetilde{v}^{s*}_{t+1}+\widetilde{d}^{s*}_{t+1}}{\widetilde{v}^{s*}_t}\frac{Q_{t+1}}{Q_t}.$$

These returns on equities are adjusted by $1-\delta = N_{D,t+1}/\left(N_{D,t}+N_{E,t}\right) = N^*_{D,t+1}/\left(N^*_{D,t}+N^*_{E,t}\right)$, the surviving rate of producing firms and entrants between the two time periods.

Other than equities, the household holds bonds defined in terms of a domestic consumption basket. Letting $b_{h,t+1}$ and $b^*_{f,t+1}$ be these bond holdings and returns on bonds be r_{t+1} and r^*_{t+1} at time t into $t+1$ gives the following gross returns between t and $t+1$ (in units of Home consumption):

$$R^b_{h,t+1} \equiv 1+r_{t+1},\quad R^b_{f,t+1} \equiv \left(1+r^*_{t+1}\right)\frac{Q_{t+1}}{Q_t}.$$

The budget constraint has labor and financial income as revenue and has consumption, equity and bond holdings as expenditures. The period budget constraint of the representative household in Home (defined in units of Home consumption) is given by[2]

$$C_t + \widetilde{v}^s_t\left(N_{D,t}+N_{E,t}\right)s_{h,t+1} + b_{h,t+1} = w_t L_t + R^s_{h,t}\widetilde{v}^s_{t-1}\left(N_{D,t-1}+N_{E,t-1}\right)s_{h,t} + R^b_{h,t}b_{h,t}. \quad (2)$$

[2]The corresponding budget constraint for Foreign households is

The representative household maximizes the expected intertemporal utility with respect to $s_{h,t+1}$, $b_{h,t+1}$, L_t and C_t subject to (2) for all periods. As a result, Euler equations for share holdings can be derived as[3]

$$1 = \beta E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} R^s_{h,t+1} \right].$$

Euler equations for bond holdings are given by[4]

$$1 = \beta E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} R^b_{h,t+1} \right].$$

Finally, the optimal labor supply is given by

$$\chi \left(L_t \right)^{\frac{1}{\psi}} = w_t C_t^{-\gamma}.$$

2.7 General Equilibrium and Balanced Trade

Supplied labor units L_t are demanded for fixed costs of exporting and firm creation and for the production of domestic and tradable goods, which implies that

$$L_t = \frac{N_{E,t} \widetilde{v}_t^s}{w_t} + \frac{(\sigma - 1) N_{D,t} \widetilde{d}_t}{w_t} + \frac{\sigma N_{X,t} f_X}{Z_t}.$$

$$C_t^* + \widetilde{v}_t^{s*} \left(N^*_{D,t} + N^*_{E,t} \right) s^*_{f,t+1} + b^*_{f,t+1}$$
$$= w_t^* L_t^* + \frac{Q_t}{Q_{t+1}} R^s_{f,t} \widetilde{v}^{s*}_{t-1} \left(N^*_{D,t-1} + N^*_{E,t-1} \right) s^*_{f,t} + \frac{Q_t}{Q_{t+1}} R^b_{f,t} b^*_{f,t} \quad (1)$$

[3]Those for the Foreign representative household become

$$1 = \beta E_t \left(\frac{C^*_{t+1}}{C^*_t} \right)^{-\gamma} R^s_{f,t+1} \frac{Q_t}{Q_{t+1}}.$$

[4]Those of Foreign counterparts are

$$1 = \beta E_t \left[\left(\frac{C^*_{t+1}}{C^*_t} \right)^{-\gamma} R^b_{f,t+1} \frac{Q_t}{Q_{t+1}} \right].$$

The model is completed by considering the balanced trade condition such that

$$\int_0^{N^*_{X,t}} p^*_{X,t}\left(\vartheta^*\right) c_{X,t}\left(\vartheta^*\right) d\vartheta^* = \int_0^{N_{X,t}} p_{X,t}\left(\vartheta\right) c^*_{X,t}\left(\vartheta\right) d\vartheta.$$

Using the demand system found previously, this is equivalent to

$$N_{X,t}^{\psi(\sigma-1)}\left(\frac{\widetilde{\rho}_{X,t}}{\widetilde{q}_{X,t}}\right)^{1-\sigma}\rho_{H,t}^{*\sigma-\omega}Q_t C_t^* = N_{X,t}^{*\psi(\sigma-1)}\left(\frac{\widetilde{\rho}^*_{X,t}}{\widetilde{q}^*_{X,t}}\right)^{1-\sigma}\rho_{F,t}^{\sigma-\omega}C_t. \tag{3}$$

The whole system including the steady state is summarized in Table 2

2.8 Calibration

We calibrate the theoretical models using parameter values, as in Table 3. The calibration is conducted on a quarterly basis. The values of constant risk aversion (γ), the steady-state discount factor (β), the Frisch elasticity of the labor supply (φ) and the elasticity of substitution between local goods and imported goods (ω) are in line with the literature on open macroeconomics. The value of the death shock (δ), the elasticity of substitution among product varieties (σ), the preference for variety (ψ), fixed export costs (f_X) and the shape of the Pareto distribution (k) are set following Ghironi and Melitz (2005). These values are based on the empirical findings of Bernard et al. (2003), which also document that the proportion of exporting firms is 21%. The value of fixed export costs is taken such that in the steady state, the share of exporters is 21% accordingly. The parameter that determines the quality ladder in the economy (ϕ) comes from Feenstra and Romalis (2014), who estimate the elasticity of firm-specific quality with respect to firm-specific productivity using global trade data.

The productivity process are selected from Backus et al. (1992) such that $\mathsf{Z}_{t+1}=\Omega\mathsf{Z}_t+\xi_t$, where $\mathsf{Z}_t = \begin{bmatrix} Z_t, & Z^*_t \end{bmatrix}'$, $\xi_t = \begin{bmatrix} \xi_t, & \xi^*_t \end{bmatrix}'$ and

$$\Omega = \begin{bmatrix} 0.906 & 0.088 \\ 0.088 & 0.906 \end{bmatrix}, \text{ and } V\left(\xi\right) = \begin{bmatrix} 0.73 & 0.19 \\ 0.19 & 0.73 \end{bmatrix}.$$

where ξ_t is assumed to be zero mean i.i.d..

Table 2: The Model

Price indices	$\rho_{H,t}^{1-\omega}+\rho_{F,t}^{1-\omega}=1,\ \rho_{H,t}=N_{D,t}^{-\psi}\frac{\widetilde{\rho}_{D,t}}{\widetilde{q}_D},\ \rho_{F,t}=N_{X,t}^{*-\psi}\frac{\widetilde{\rho}_{X,t}^*}{\widetilde{q}_{X,t}^*}$
	$\rho_{F,t}^{*1-\omega}+\rho_{H,t}^{*1-\omega}=1,\ \rho_{F,t}^*=N_{D,t}^{*-\psi}\frac{\widetilde{\rho}_{D,t}^*}{\widetilde{q}_D^*},\ \rho_{H,t}^*=N_{X,t}^{-\psi}\frac{\widetilde{\rho}_{X,t}}{\widetilde{q}_{X,t}}$
Pricing	$\widetilde{\rho}_{D,t}=\frac{\sigma}{\sigma-1}\frac{1}{1-\phi}\frac{w_t}{Z_t\widetilde{z}_D},\quad \widetilde{\rho}_{X,t}=\tau_t\frac{\sigma}{\sigma-1}\frac{1}{1-\phi}\frac{w_t}{Z_t\widetilde{z}_{X,t}}Q_t^{-1},$
	$\widetilde{\rho}_{D,t}^*=\frac{\sigma}{\sigma-1}\frac{1}{1-\phi}\frac{w_t}{Z_t^*\widetilde{z}_D^*},\quad \widetilde{\rho}_{X,t}^*=\tau_t\frac{\sigma}{\sigma-1}\frac{1}{1-\phi}\frac{w_t^*}{Z_t^*\widetilde{z}_{X,t}^*}Q_t$
Profits	$\widetilde{d}_t=\widetilde{d}_{D,t}+\frac{N_{X,t}}{N_{D,t}}\widetilde{d}_{X,t},\quad \widetilde{d}_{D,t}=\frac{1}{\sigma}N_{D,t}^{\psi(\omega-1)-1}\left(\frac{\widetilde{\rho}_{D,t}}{\widetilde{q}_D}\right)^{1-\omega}C_t$
	$\widetilde{d}_{X,t}=\frac{Q_t}{\sigma}N_{X,t}^{\psi(\omega-1)-1}\left(\frac{\widetilde{\rho}_{X,t}}{\widetilde{q}_{X,t}}\right)^{1-\omega}C_t^*-\frac{w_t f_X}{Z_t}$
	$\widetilde{d}_t^*=\widetilde{d}_{D,t}^*+\frac{N_{X,t}^*}{N_{D,t}^*}\widetilde{d}_{X,t}^*,\quad \widetilde{d}_{D,t}^*=\frac{1}{\sigma}N_{D,t}^{*\psi(\omega-1)-1}\left(\frac{\widetilde{\rho}_{D,t}^*}{\widetilde{q}_D^*}\right)^{1-\omega}C_t^*$
	$\widetilde{d}_{X,t}^*=\frac{Q_t^{-1}}{\sigma}N_{X,t}^{*\psi(\omega-1)-1}\left(\frac{\widetilde{\rho}_{X,t}^*}{\widetilde{q}_{X,t}^*}\right)^{1-\omega}C_t-\frac{w_t^* f_X^*}{Z_t^*}$
Free entry	$\widetilde{v}_t^s=\frac{w_t}{Z_t}f_E,\qquad \widetilde{v}_t^{s*}=\frac{w_t^*}{Z_t^*}f_E^*$
LMC	$w_tL_t=N_{E,t}\widetilde{v}_t^s+(\sigma-1)\,N_{D,t}\widetilde{d}_t++\sigma N_{X,t}\frac{w_t f_X}{Z_t}$
	$w_t^*L_t^*=N_{E,t}^*\widetilde{v}_t^{s*}+(\sigma-1)\,N_{D,t}^*\widetilde{d}_t^*+\sigma N_{X,t}^*\frac{w_t^* f_X^*}{Z_t^*}$
Export share	$\frac{N_{X,t}}{N_{D.t}}=z_{\min}^k\left(\widetilde{z}_{X,t}\right)^{-k}\left[\frac{k}{k-(\sigma-1)}\right]^{\frac{k}{\sigma-1}},\quad \frac{N_{X,t}^*}{N_{D,t}^*}=z_{\min}^k\left(\widetilde{z}_{X,t}^*\right)^{-k}\left[\frac{k}{k-(\sigma-1)}\right]^{\frac{k}{\sigma-1}}$
ZCP	$\widetilde{d}_{X,t}=\frac{w_t f_X}{Z_t}\frac{\sigma-1}{k-(\sigma-1)},\quad \widetilde{d}_{X,t}^*=\frac{w_t^* f_X^*}{Z_t^*}\frac{\sigma-1}{k-(\sigma-1)}$
AEQ	$\widetilde{q}_{X,t}=\left(\frac{\phi}{1-\phi}\widetilde{z}_{X,t}\right)^{\phi},\ \widetilde{q}_{X,t}^*=\left(\frac{\phi}{1-\phi}\widetilde{z}_{X,t}^*\right)^{\phi}$
Number of firms	$N_{D,t+1}=(1-\delta)\,(N_{D,t}+N_{E,t}),\quad N_{D,t+1}^*=(1-\delta)\left(N_{D,t}^*+N_{E,t}^*\right)$
Euler shares	$1=\beta E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}R_{h,t+1}^s\right]$
	$1=\beta E_t\left[\left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\gamma}R_{f,t+1}^s\frac{Q_t}{Q_{t+1}}\right]$
Euler bonds	$1=\beta E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}R_{h,t+1}^b\right]$
	$1=\beta E_t\left[\left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\gamma}R_{f,t+1}^b\frac{Q_t}{Q_{t+1}}\right]$
Balanced trade	$N_{X,t}^{\psi(\sigma-1)}\left(\frac{\widetilde{\rho}_{X,t}}{\widetilde{q}_{X,t}}\right)^{1-\sigma}\rho_{H,t}^{*\sigma-\omega}Q_tC_t^*=N_{X,t}^{*\psi(\sigma-1)}\left(\frac{\widetilde{\rho}_{X,t}^*}{\widetilde{q}_{X,t}^*}\right)^{1-\sigma}\rho_{F,t}^{\sigma-\omega}C_t$

3 International Risk Sharing with Quality and Variety

In this section, we explore the nature of international risk sharing, namely, the Backus-Smith puzzle. Although under the balanced trade condition, there is no possibility of

Table 3: Baseline Parameter Values

γ	constant risk aversion	2
β	discount factor	0.99
φ	Frisch elasticity of labor supply	2
σ	elasticity of substitution among varieties	3.8
ω	between Home and Foreign goods	2
τ	steady state trade cost	1.3
δ	death shock	0.025
k	Pareto distribution	3.34
ψ	Preference for variety	Dixit-Stiglitz
ϕ	quality ladder	0.61

international borrowing and lending, it is shown that consumption risk can be insured to some extent through appropriate fluctuations in product quality and the number of product varieties, as well as the international relative price of export goods. In addition, we show that imperfectly observable fluctuations in product quality and the number of varieties drive a wedge between the observable relative consumption and the observable real exchange rate. We show that structurally, the BS correlation is conditional on fluctuations in the number of product varieties and product quality.

3.1 The Backus-Smith Puzzle

Why is the consumption growth rate different across countries? If there exists a system of perfect consumption risk sharing, we should expect to see a strong comovement of consumption growth across countries despite idiosyncratic income shock. This predicted pattern under complete asset markets (and the low level of one price) is strongly rejected with actual data[5] By explicitly considering the difference in the price level of consumption goods across countries, Kollmann (1995) and Backus and Smith (1993) derive a general-

[5]This is the well-known consumption growth rate puzzle. See Backus et al. (1992) and Obstfeld and Rogoff (2000).

ization of the above anomaly. Namely, under complete asset markets, the marginal utility expressed in terms of the same consumption unit is equalized across countries:

$$U_{C,t} = U_{C^*,t} Q_t$$

With our CRRA utility function, we have

$$\mathsf{C} - \mathsf{C}^* = \frac{1}{\gamma} \mathsf{Q} \tag{4}$$

In the above expressions and henceforth, Sans Serif font denotes the first-order deviations. We have also dropped time indices for the sake of simplicity, as we know that the expressions hold for all time periods. Since $\gamma \geq 1$, as argued in Corsetti et al. (2010) and others, the correlation between relative consumption and the real exchange rate should be positive. This is, however, not the case in reality. The correlation between the relative consumption and the real exchange rate is close to zero or even negative, which is known as the Backus-Smith puzzle.

3.2 The Backus-Smith Relation with Product Quality and Variety

3.2.1 The Welfare-Based Backus-Smith Relation

The failure of systematic positive comovement between relative consumption and the real exchange rate fluctuations that we expect to see under complete markets (4) would be attributed to the assumption of complete asset markets itself (Obstfeld and Rogoff 2000). When the households no longer have access to this international financial market, what would happen to the relationship between cross-country consumption and real exchange rate fluctuations? As is the case of our benchmark model, by removing any possibilities of risk sharing with internationally held financial assets, the relationship between the relative consumption across countries and the real exchange rate fluctuations can be expressed as follows[6]

$$\mathsf{C}-\mathsf{C}^{*}=\frac{2S_{ED}\omega-1}{2S_{ED}-1}\mathsf{Q}+\frac{(\omega-1)\,S_{ED}}{2S_{ED}-1}\left[\psi(\mathsf{N}_{D}^{R}-\mathsf{N}_{X}^{R})+(1+\frac{1}{\phi})\widetilde{\mathsf{q}}_{X}^{R}\right]. \tag{5}$$

where $\mathsf{N}_{D}^{R}=\mathsf{N}_{D}-\mathsf{N}_{D}^{*}$, $\mathsf{N}_{X}^{R}=\mathsf{N}_{X}-\mathsf{N}_{X}^{*}$, $\widetilde{\mathsf{q}}_{X}^{R}=\widetilde{\mathsf{q}}_{X}-\widetilde{\mathsf{q}}_{X}^{*}$ and that S_{ED} ($>1/2$) is the steady-state expenditure share on domestically produced goods. The above equation determines the welfare-based fluctuations in relative consumption and the real exchange rate together with changes in the number of product varieties and quality.[7]

Theorem 1. *In the world where product quality and the number of product varieties fluctuate endogenously, the equilibrium allocation under financial autarky (5) perfectly mimics that obtained with complete asset markets (4) when* $\omega=\gamma=1$.

In a simple endowment economy without quality and variety, Cole and Obstfeld (1991) argue that even without any kind of cross-border asset holdings, perfect international risk sharing can be achieved through appropriate fluctuations in relative prices. Our paper hence generalizes their results with changes in the number of product varieties and product quality.

To precisely observe the transmission mechanism, note that we can express the fluctuations in the welfare-based real exchange rate, Q, as

$$\mathsf{Q}=(2S_{ED}-1)\,\mathsf{TOL}+\psi S_{ED}\mathsf{N}_{D}^{R}-(1-S_{ED})\left[\psi\mathsf{N}_{X}^{R}+(1+\frac{1}{\phi})\widetilde{\mathsf{q}}_{X}^{R}\right], \tag{6}$$

[6] In our model, using the demand system and cutoff quality changes that follow cutoff productivity changes in the export market, the balanced trade condition (3) can be expressed as

$$\omega\mathsf{Q}-(\mathsf{C}-\mathsf{C}^{*})+\psi\,(\omega-1)\,\mathsf{N}_{X}^{R}-(\omega-1)\left[\mathsf{w}^{R}-\mathsf{Z}^{R}-(1+\frac{1}{\phi})\widetilde{\mathsf{q}}_{X}^{R}\right]=0.$$

Plugging the decomposition of the real exchange rate (6) into the above balanced trade condition, we get (5).

[7] By comparing the expression found in Hamano (2013) that has only the first term on the right-hand side of the equation (5), the second term in the square brackets arises due to the presence of exporting fixed costs. By setting $f_X=f_X^{*}=0$, all firms export independent of their specific productivity. As a result, we do not see any changes in cutoff and quality as $\widetilde{\mathsf{z}}_{X}^{R}=\widetilde{\mathsf{q}}_{X}^{R}=0$ and the number of exporters and domestic producers coincide as $\mathsf{N}_{D}^{R}=\mathsf{N}_{X}^{R}$.

where $\mathsf{TOL} = -\left(\mathsf{w}^R - \mathsf{Z}^R\right)$ in which $\mathsf{w}^R = \mathsf{w} - \mathsf{w}^*$ and $\mathsf{Z}^R = \mathsf{Z} - \mathsf{Z}^*$. The first term on the right-hand side of the equation, TOL, is the *terms of labor* (Ghironi and Melitz, 2005), which measures the relative cost of effective units of labor across countries. The second term represents the welfare gains arising from a relatively higher number of domestically available varieties. The last term captures changes in the relative number of export goods and the relative cutoff changes in the quality of these goods across countries. Real appreciation for Home (fall in Q) occurs following an appreciation in the terms of labor, a relatively lower number of domestically available varieties and a lower number of available varieties from abroad, as well as lower product quality.[8]

3.2.2 The Empirical Based Backus-Smith Relation

Before we fully explore the Backus-Smith puzzle with product quality and variety, one additional issue must be addressed: as argued in (5), consumption and the real exchange rate are measured only imperfectly with respect to changes in the number of varieties and product quality (Broda and Weinstain, 2004, 2006). We address this point properly.

In capturing changes in product quality and the number of varieties, there might be some differences across statistical agency and across time periods in terms of their accuracy. Accordingly, we characterize this aspect in a general way. Specifically, we define the empirically observable fluctuations in the real exchange rate, $\widehat{\mathsf{Q}}$, as

$$\begin{aligned}\widehat{\mathsf{Q}} = \mathsf{Q} - \psi\lambda_1\mathsf{N}_D^R + \psi\lambda_2\mathsf{N}_X^R + \lambda_3\widetilde{\mathsf{q}}_X^R \\ = (2S_{ED} - 1)\,\mathsf{TOL} + \psi(S_{ED} - \lambda_1)\mathsf{N}_D^R \\ - \psi\,(1 - S_{ED} - \lambda_2)\,\mathsf{N}_X^R - \left[(1 - S_{ED})\,(1 + \frac{1}{\phi}) - \lambda_3\right]\widetilde{\mathsf{q}}_X^R \quad (7)\end{aligned}$$

where the parameter λ_1, λ_2 and λ_3 represent the degree of (in)efficiency of statistical agents in capturing changes in the number of domestic varieties N_D^R, export (import) varieties N_X^R and product quality $\widetilde{\mathsf{q}}_X^R$.[9] Depending on the value of these parameters, the

[8] See also Ghironi and Melitz (2005) and Hamano (2015) for a similar decomposition.

[9] Based on a similar idea, Aghion et al. (2017) recently argued the "missing growth" due to a systematic error of statistical agencies in capturing creative destruction and resulting in quality upgrades.

definition of $\widehat{\mathsf{Q}}$ is different. On one hand, when $\lambda_1 = \lambda_2 = \lambda_3 = 0$, there is no discrepancy between the welfare-based measures and the empirically based measures. Hence, the statistical agency is perfectly efficient: $\widehat{\mathsf{Q}} = \mathsf{Q}$. On the other hand, when $\lambda_1 = S_{ED}$ and $\lambda_2 = \lambda_3 = 1 - S_{ED}$, the statistical agency completely ignores fluctuations in the number of varieties and the product quality.[10] In general, when $\lambda_1 > 0$ ($\lambda_1 < 0$), the statistical agents under (over) estimate the impact of domestic varieties in the consumption basket. In a similar way, when $\lambda_2 > 0$ ($\lambda_2 < 0$) and $\lambda_3 > 0$ ($\lambda_3 < 0$), they under (over) estimate the impact of import varieties and product quality. In a similar way, we can define the empirically based fluctuation in the relative consumption as

$$\widehat{\mathsf{C}} - \widehat{\mathsf{C}}^* = \mathsf{C} - \mathsf{C}^* - \psi\lambda_1 \mathsf{N}_D^R + \psi\lambda_2 \mathsf{N}_X^R + \lambda_3 \widetilde{\mathsf{q}}_X^R. \tag{8}$$

Finally, using the above definition of the empirically based fluctuations (7) and (8), we can rewrite the welfare-based relation (5) as

$$\begin{aligned} \widehat{\mathsf{C}} - \widehat{\mathsf{C}}^* = & \frac{2S_{ED}\omega - 1}{2S_{ED} - 1}\widehat{\mathsf{Q}} + \frac{\psi\left(2\lambda_1 - 1\right)\left(\omega - 1\right)S_{ED}}{2S_{ED} - 1}\mathsf{N}_D^R \\ & - \frac{\psi\left(2\lambda_2 - 1\right)\left(\omega - 1\right)S_{ED}}{2S_{ED} - 1}\mathsf{N}_X^R + \frac{\left(\omega - 1\right)S_{ED}}{2S_{ED} - 1}\left[\frac{1}{\phi} - \left(2\lambda_3 - 1\right)\right]\widetilde{\mathsf{q}}_X^R \end{aligned} \tag{9}$$

The equation (9) is the empirical counterpart of the welfare-based relation (5).[11] The first term is basically the same term argued in Corsetti et al. (2008) in the absence of changes in the number of product varieties and quality. Importantly, the observable correlation between cross-country differences in consumption growth and real exchange rate fluctuations is *conditional* on unobservable changes in product quality and the number of product varieties. This conditional correlation is unambiguously positive since $\left(2S_{ED}\omega - 1\right)/\left(2S_{ED} - 1\right) > 0$ with the conventional value of the elasticity of substitution such as $\omega > 1$. However, the *unconditional* correlation between the relative consumption and the real exchange rate can be positive or negative depending on structural parameters in the economy. The signs on the number of domestic varieties N_D^R, export (import)

[10]This is the case in Corsetti et al. (2007).

[11]Again, when $\lambda_1 = \lambda_2 = \lambda_3 = 0$, the expression (9) coincides to (5). In particular, by removing the fixed cost for exporting and setting $\lambda_1 = S_{ED}$ and $\lambda_2 = 1 - S_{ED}$, the expression (9) becomes identical to the one found in Hamano (2013) with changes only in the number of product varieties.

varieties N_X^R and the product quality $\widetilde{\mathsf{q}}_X^R$ depend on the value of parameters λ_1, λ_2 and λ_3, which represent the (in)efficiency of statistical agents.[12]

What do we find quantitatively about the BS correlation in our theoretical model? In Figure 1, we document the empirically based measure of relative consumption across countries (solid line) and the real exchange rate (dashed line) following a 1% productivity rise in Home. Here, we remove the international spillover in the productivity process for clarification purposes. In specifying the theoretical counterpart to the empirical measure, we use a similar (in)efficiency value of parameters as in Feenstra (1994) and Ghironi and Melitz (2005) such that $\lambda_1 = \lambda_2 = \lambda_3 = 1$.

As in the figure, following a positive shock in Home, the relative consumption increases and the real exchange rate appreciates sharply after the appreciation in the terms of labor. Here, the appreciation of the real exchange rate is thus driven by a more elaborate Harrod-Balassa-Samuelson type mechanism that allows changes in product quality and the number of export varieties based on heterogeneous firms.[13] With the benchmark calibration, the BS correlation is 0.16.[14]

How is the BS correlation sensitive with respect to the quality ladder and the (in)efficiency of statistical agencies? In Figure 2, we provide a sensitivity analysis against the value of the quality ladder, ϕ. In addition to the benchmark calibration (solid line), the figure documents an unconditional BS correlation with an alternative empirically based measure such that $\lambda_1 = S_{ED}$, $\lambda_2 = \lambda_3 = 1 - S_{ED}$, given less underestimation (dashed line).

[12] Note that in our model, it is possible to break the tight link we see under complete financial markets (4) as

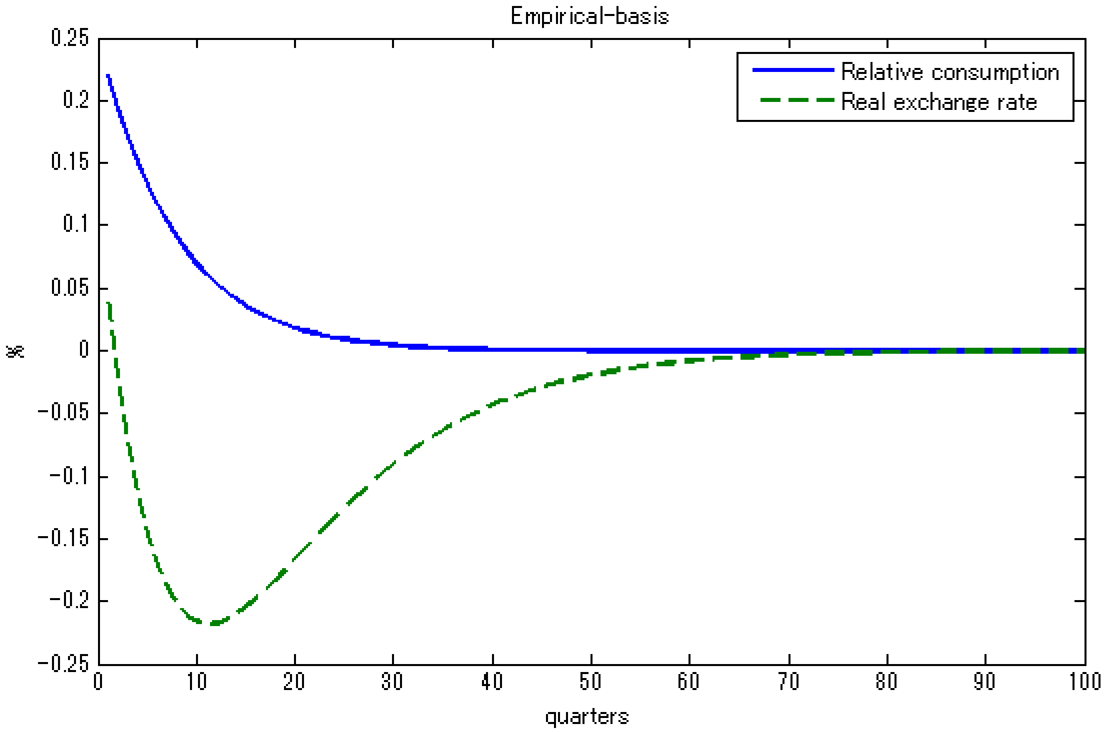
$$\widehat{\mathsf{C}}-\widehat{\mathsf{C}}^*=\frac{1}{\gamma}\widehat{\mathsf{Q}} - \left(1-\frac{1}{\gamma}\right)\left\{\psi\left[S_{ED}\lambda_1\mathsf{N}_D^R-(1-S_{ED})\lambda_2\mathsf{N}_X^R)\right]-(1-S_{ED})\lambda_3\widetilde{\mathsf{q}}_X^R\right\}.$$

Product quality and variety work as a preference shock. See Stockman and Tesar (1995), Raffo (2010) and Mandelman et al. (2011).

[13] Ghironi and Melitz (2005) argue the Harrod-Balassa-Samuelson effect arising from endogenous nontradeness based on heterogeneous firms in an expression similar to (7) but without quality changes.

[14] On a welfare basis, the unconditional BS correlation is negative and amounts to −0.65. In our calibration, the welfare-based real exchange rate appreciates even with a rise in domestically available varieties in Home compared to Foreign.

Figure 1: Observable Relative Consumption and Real Exchange Rate Fluctuations



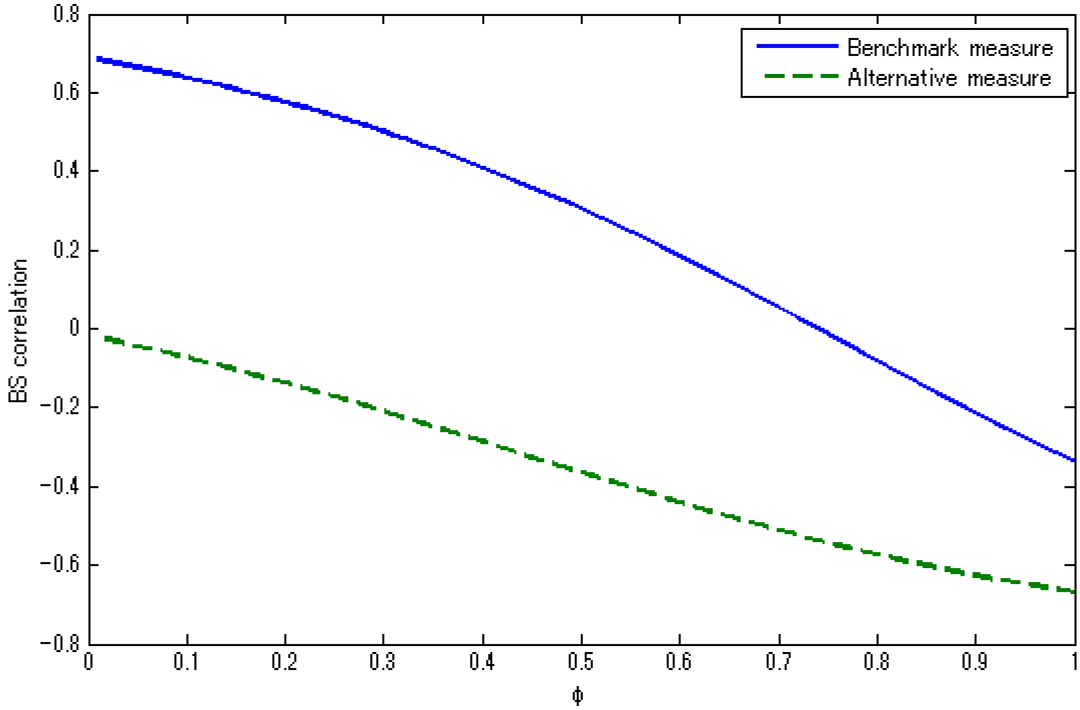
Note: The figure reports the deviation from the steady state of relative consumption across countries $\widehat{C}_t/\widehat{C}_t^*$ and the real exchange rate $\widehat{Q}_t$ (both are empirically consistent measures) following a 1 % rise in Home productivity Z_t without cross-country spillover of the productivity process.

It is observed that as ϕ increases from zero, the BS correlation changes from positive to negative in the benchmark calibration, and a similar pattern is observed for the alternative calibration. A higher value of ϕ implies more competition in terms of quality, which requires labor as an input. Following a rise in the labor demand, wages increase, which further appreciates the terms of labor. Such a sharp appreciation in the terms of labor results in a sharper appreciation in the empirically based real exchange rate $\widehat{Q_t}$, demonstrating a stronger negative BS correlation.

3.3 International Risk Sharing with State Non-Contingent Bonds

We also show the sensitivity of our results to the specific assumption on financial markets. In Appendix A, we provide a model with state non-contingent bonds with which households insure their consumption risk only after the realization of productivity shock.

Figure 2: The Backus-Smith Correlation and Quality Ladder (Balanced Trade)



Note: The figure reports the sensitivity result of the unconditional BS correlation in the theoretical model against the quality ladder, ϕ, with the benchmark measurement error ($\lambda_1 = \lambda_2 = \lambda_3 = 1$) and the alternative measurement error ($\lambda_1 = S_{ED}$, $\lambda_2 = \lambda_3 = 1 - S_{ED}$) obtained under balanced trade.

It is well known that with internationally held state non-contingent bonds, the expected growth rate of the real exchange rate is equal to the expected growth rate in relative consumption:

$$E_t\left[(\mathsf{C}_{t+1}-\mathsf{C}_t) - \left(\mathsf{C}^*_{t+1}-\mathsf{C}^*_t\right)\right] \approx \frac{1}{\gamma}E_t\left(\mathsf{Q}_{t+1}-\mathsf{Q}_t\right), \tag{10}$$

where we abstract away from negligible fluctuations in bond holdings arising from quadratic adjustment costs. As is well known, with this specification, some of the shock can be insured *ex post* with financial assets. With the benchmark values of parameters, the BS correlation is -0.18. Figure 5 provides the result of the sensitivity analysis of the BS correlation with respect to quality ladder ϕ. We can observe a similar case with balanced trade.[15]

[15]The welfare–based BS correlation is -0.51 in our calibration. The impulse response functions of other variables with non-contingent bond holdings following a transitory productivity shock are available upon request.

Corsetti et al. (2008) points out the role played by shock persistence in generating a realistic BS correlation in this environment. Higher income anticipated in the future due to higher persistence increases wealth today, providing a sharper appreciation in the terms of labor in transitory dynamics. Figure 6 gives the result of a sensitivity analysis with respect to the shock persistence of productivity shock. We confirm the wealth effect due to high shock persistence.

4 Empirical Investigation

4.1 The Backus-Smith Correlation with Data

As argued in the previous section, the structural relationship between relative consumption across countries and the observed real exchange rate is conditional on changes in product quality and the number of product varieties. In this section, we first compute the conditional Backus-Smith correlation using panel data for advanced economies. We next perform regression analysis and observe how an unconditional BS relation is different from a conditional BS relation with changes in quality and the number of varieties.

4.1.1 Data

For this purpose, we construct a panel dataset of 26 advanced economies. Feenstra and Romalis (2014) provide a dataset of their estimates of quality of exports and imports for each good (defined in four-digit SITC codes) for each country in the world for the period from 1984 to 2011. Their estimates of product quality are defined with respect to the quality of the ROW, whose average is normalized to unity. Based on their estimates, we compute the annual time series of the aggregate quality of exports and imports for all countries in the sample. Specifically, based on the estimated quality of a particular good j of export ($s = X$) or import ($s = M$) of a country i for a year t, q^i_{jst}, the aggregated quality of that country's exports or imports for that year is computed by

$$q^i_{st} = \sum^{N^i_{st}} ts^i_{jst} q^i_{jst}$$

where N^i_{st} is the number of exported or imported varieties (or precisely, the number of categories of goods defined in terms of four-digit SITC codes) with the ROW, and ts^i_{jst} is the share of exports or imports of that particular good j.

In Appendix C, we present the descriptive statistics and the evolution of the number of varieties of exports and imports, as well as their quality, in our sample. The average number of export varieties (categories of goods) amounts to 740, while that of import varieties is 696.1 for each year. The average quality of export products is 1.2 and that of import products is 1.1. These relatively high numbers reflect that there are only advanced economies in our sample. We see a large drop for the number of both export and import varieties beginning in the year 2009, the time of the "great trade collapse" following the world financial crisis, while the variations in the quality of exports and imports are relatively limited. In the companion paper, Hamano (2016) analyzes the trade dynamics of product quality and the number of varieties in more detail.

The data for the real exchange rate come from the narrow indices of the real effective exchange rate provided by BIS. Our choice of countries corresponds to the BIS's definition in computing the narrow indices.[16] The data of real per capita consumption and per capita income come from the most recent Penn World Table (pwt90).

4.1.2 Unconditional vs. Conditional Backus-Smith Correlation with Quality and Variety

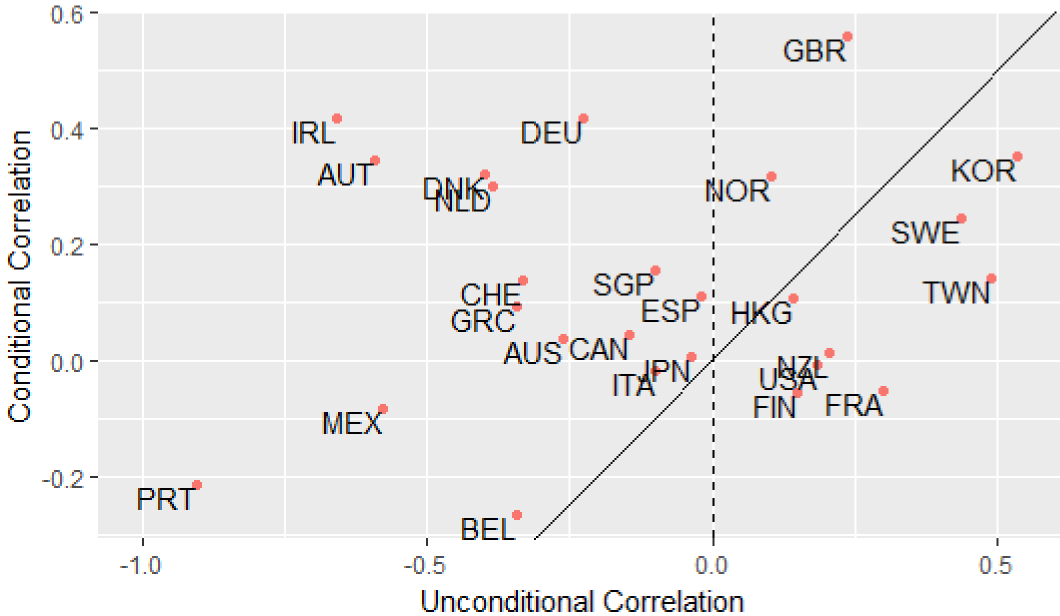
Here, we compare two types of BS correlations: unconditional correlation and conditional correlation on changes in the number of varieties and their quality. Specifically, we compute both unconditional and conditional correlations between the consumption growth rate and the growth rate of the real exchange rate for each country with respect to the world average (average among all countries in the sample).

The consumption growth rate of country i, ΔC^i_t, and its world average, ΔC^W_t, are defined as $\Delta C^i_t = \ln C^i_t - \ln C^i_{t-1}$and $\Delta C^W_t = \ln C^W_t - \ln C^W_{t-1}$.[17] The growth rate of the

[16]These are AUS, AUT, BEL, CAN, CHE, DEU, DNK, ESP, FIN, FRA, GBR, GRC, HKG, IRL, ITA , JPN, KOR, MEX, NLD, NOR, NZL, PRT, SGP, SWE, TWN and USA as in Table 1.

[17]Income growth rate of country i and the world average are defined in a similar way.

Figure 3: Unconditional vs. Conditional Backus-Smith Correlation

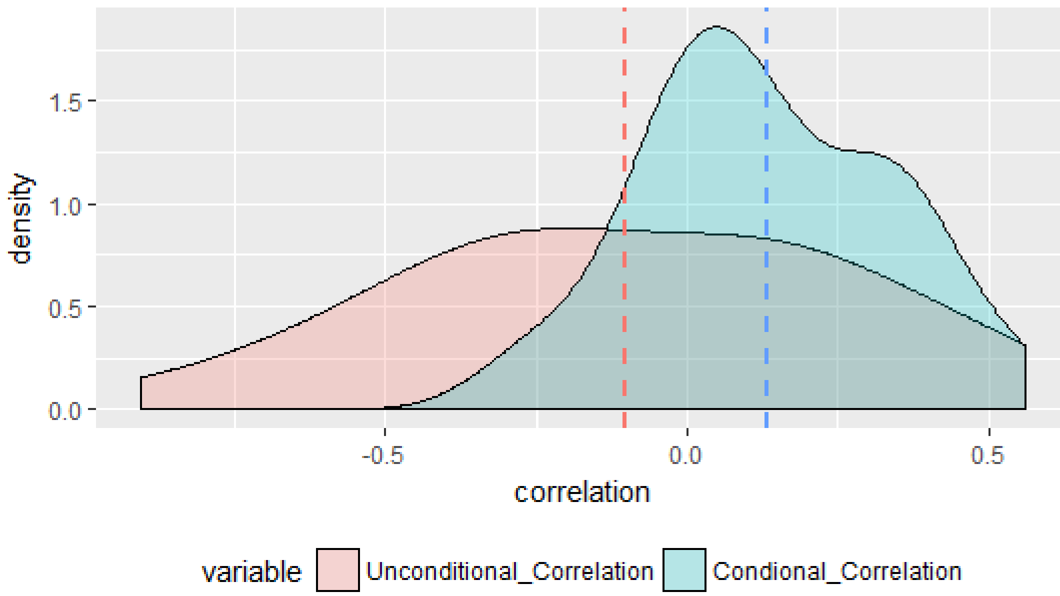


Note: Unconditional and conditional Backus-Smith correlations are plotted for each advanced economy for the period from 1984 to 2011, together with a 45-degree line. Our sample includes 26 advanced economies where Hong Kong, Singapore and Mexico are not OECD countries.

real effective exchange rate is defined as $\Delta REER_t^i = -(\ln REER_t^i - \ln REER_{t-1}^i)$. The growth rate of the number of export or import varieties and that of quality are defined as $\Delta N_{s,t}^i = \ln N_{s,t}^i - \ln N_{s,t-1}^i$ and $\Delta q_{s,t}^i = \ln q_{s,t}^i - \ln q_{s,t-1}^i$, respectively, where $s = X$ (Export) or M (Import). Note that the effective real exchange rate growth, the number of varieties and product quality are already defined with respect to the world average (ROW).

We then compute two types of correlations: unconditional and conditional correlations, namely, $Corr(\Delta C_t^i - \Delta C_t^W,\ \Delta REER_t^i)$ and $Corr(\Delta C_t^i - \Delta C_t^W,\ \Delta REER_t^i \mid \Delta N_{X,t}^i - \Delta N_{M,t}^i,\ \Delta q_{X,t}^i - \Delta q_{M,t}^i)$ for each country during the entire sample period. We plot the unconditional BS correlation on the horizontal axis and the conditional correlation on the vertical axis, together with a 45-degree line in Figure 3. We see close to zero or even negative unconditional BS correlation for a large number of countries. It is striking to see that the correlations increase once these are conditioned on changes in the number of varieties and quality. Indeed, 18 countries among 26 are situated above the 45 degree line and therefore “improve” the extent of consumption risk sharing.

Figure 4: Distribution of the Backus-Smith Correlations



Note: The distribution of both unconditional and conditional Backus-Smith correlations are reported. The mean for the unconditional BS correlation is -0.10, while that of the conditional correlation is 0.13.

Figure 4 shows the distribution of the correlations. The mean for the unconditional BS correlation is -0.10, while that of the conditional correlation is 0.13. A simple t-test on the difference of these means is significant at more than 1 % level. Also note that the variance in the correlation declines with the conditional measure, indicating a uniform improvement in risk sharing for all countries.

4.1.3 Regression Analysis

Additionally, we investigate the implication of conditioning BS relations with variety and quality with a panel regression analysis. Based on the structural relation (9), the benchmark equations we test are as follows:

$$\Delta C_t^i - \Delta C_t^W = \beta_0 + \beta_1 \Delta REER_t^i + \mu^i + \nu_t + \xi_t^i, \tag{11}$$

$$\Delta C_t^i - \Delta C_t^W = \beta_0 + \beta_1 \Delta REER_t^i + \beta_2 \left(\Delta N_{X,t}^i - \Delta N_{M,t}^i\right) + \beta_3 \left(\Delta q_{X,t}^i - \Delta q_{M,t}^i\right) + \mu^i + \nu_t + \xi_t^i, \tag{12}$$

where μ^i and ν_t represent country-specific and time fixed effects, respectively. ξ_t^i denotes i.i.d. shock.

Table 4 shows the results of our estimation using pooled, fixed and random effects. As shown, by conditioning the BS relation with changes in the number of varieties and quality, the BS coefficients are significantly positive (β_1 of equation (12)), while without conditioning, these are not significant (β_1 of equation (11)). By conditioning, the BS coefficients increase, indicating a resolution of the puzzle and better risk sharing across countries. Note also that the sign of the coefficient of the number of varieties and quality are also consistent with the theoretical relation (9) with our benchmark value of parameters throughout all specifications.

For a further robustness check, we also include the relative income growth rate $\Delta Y_t^i - \Delta Y_t^W$ as an explanatory variable, as in Kose et al. (2009) and Hess and Shin (2010). This can also be considered as a proxy of relative changes in the domestic number of varieties in the theoretical relation (9). As we can see in Table 5, the income growth rate is highly significant in explaining consumption growth, and the BS coefficients become less puzzling. By conditioning the BS relation with changes in the number of traded varieties and quality, the BS coefficients further improve for all specifications, as is the case in the benchmark empirical specification.

5 Conclusion

This paper provides a simple two-country DSGE model that captures both changes in the number of product varieties and product quality. In our theoretical model, firms that are heterogeneous in their specific productivity choose their product quality endogenously. By assuming that a higher quality of goods requires a higher cost of production, we show that quality upgrading in the economy produces a wealth effect, which generates real exchange rate appreciation and, hence, a data-consistent correlation between relative consumption and real exchange rate fluctuations.

Taking into account measurement error regarding fluctuations in the number of va-

Table 4: Backus-Smith Regression

	Dependent variable: $\Delta C_t^i - \Delta C_t^W$					
	OLS			*panel*		
	Pooled	Pooled	Fixed	Fixed	Random	Random
	(1)	(2)	(3)	(4)	(5)	(6)
$\Delta REER_t^i$	0.070	0.147***	0.058	0.121**	0.060	0.130***
	(0.061)	(0.050)	(0.055)	(0.048)	(0.055)	(0.048)
$\Delta N_{X,t}^i - \Delta N_{M,t}^i$		−1.102***		−0.803***		−0.922***
		(0.151)		(0.151)		(0.149)
$\Delta q_{X,t}^i - \Delta q_{M,t}^i$		0.603***		0.566***		0.585***
		(0.041)		(0.043)		(0.042)
Observations	680	680	680	680	680	680
R^2	0.002	0.338	0.002	0.281	0.002	0.306
Adjusted R^2	0.0005	0.336	−0.041	0.248	0.001	0.303

Note: $\Delta C_t^i - \Delta C_t^W$, $\Delta REER_t^i$, $\Delta N_{X,t}^i - \Delta N_{M,t}^i$ and $\Delta q_{X,t}^i - \Delta q_{M,t}^i$ represent the growth rate of consumption, the real exchange rate, the number of traded varieties, the quality of traded products and income for country i with ROW. In parentheses, standard errors are reported. ***, ** and * indicate significance at the 10 % 5% and 1% levels, respectively.

Table 5: Backus-Smith Regression

	Dependent variable: $\Delta C_t^i - \Delta C_t^W$					
	OLS			*panel*		
	Pooled	Pooled	Fixed	Fixed	Random	Random
	(1)	(2)	(3)	(4)	(5)	(6)
$\Delta REER_t^i$	0.058^{**}	0.074^{***}	0.065^{**}	0.081^{***}	0.060^{**}	0.076^{***}
	(0.028)	(0.028)	(0.028)	(0.028)	(0.028)	(0.028)
$\Delta N_{X,t}^i - \Delta N_{M,t}^i$		-0.187^{**}		-0.146		-0.174^{**}
		(0.088)		(0.091)		(0.088)
$\Delta q_{X,t}^i - \Delta q_{M,t}^i$		0.118^{***}		0.144^{***}		0.127^{***}
		(0.026)		(0.028)		(0.027)
$\Delta Y_t^i - \Delta Y_t^W$	0.889^{***}	0.826^{***}	0.887^{***}	0.821^{***}	0.889^{***}	0.824^{***}
	(0.018)	(0.021)	(0.021)	(0.024)	(0.018)	(0.022)
Observations	680	680	680	680	680	680
R^2	0.786	0.794	0.737	0.749	0.774	0.781
Adjusted R^2	0.785	0.793	0.725	0.737	0.773	0.780

Note: $\Delta C_t^i - \Delta C_t^W$, $\Delta REER_t^i$, $\Delta N_{X,t}^i - \Delta N_{M,t}^i$, $\Delta q_{X,t}^i - \Delta q_{M,t}^i$ and $\Delta Y_t^i - \Delta Y_t^W$ represent the growth rate of consumption, the real exchange rate, the number of traded varieties, the quality of traded products and the income for country i with ROW. In parentheses, standard errors are reported. ***, ** and * indicate significance at the 10 % 5% and 1% levels, respectively.

rieties and their quality, our theoretical model derives a structural relationship between data-consistent consumption growth across countries and data-consistent real exchange rate fluctuations. We show that the Backus-Smith relation is conditional on turnover in the number of product varieties and product quality. As a result, the negative or close-to-zero correlation that we see in the actual data is no longer puzzling. We test our model prediction and find that the BS correlation becomes positive or improves for almost all countries in our sample of major economies. A regression analysis further confirms the mechanism of the paper.

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A The Model with State Non-Contingent Bonds

In this section, we relax the assumption of financial autarky and introduce state non-contingent bonds. With this specification, net foreign assets fluctuate and countries are allowed to borrow from and lend to each other. First, the model with extension is briefly presented. Next, it is shown that the role played by product quality in international transmission and international risk sharing is qualitatively identical to that found with balanced trade.

A.1 Households

With internationally held bonds, the budget constraint of the Home representative households is

$$C_t + \widetilde{v}_t^s \left(N_{D,t} + N_{E,t}\right) s_{h,t+1} + b_{h,t+1} + Q_t b_{f,t+1} + \frac{\vartheta}{2} b_{h,t+1}^2 + \frac{\vartheta}{2} Q_t b_{f,t+1}^2$$
$$= w_t L_t + R_{h,t}^s \widetilde{v}_{t-1}^s \left(N_{D,t-1} + N_{E,t-1}\right) s_{h,t} + R_{h,t}^b b_{h,t} + R_{f,t}^b Q_{t-1} b_{f,t} + T_t^f. \quad (13)$$

With bond holdings in the budget constraint, indeterminacy in the equilibrium portfolio position and non-stationarity arise when using a linear approximation. We overcome this problem by introducing quadratic adjusting costs of bond holdings, ϑ, which guarantee a locally unique symmetric steady state with zero bond holdings and model stationarity. T_t^f is a free rebate of adjusting costs. The representative household maximizes the expected intertemporal utility with respect to $s_{h,t+1}$, $b_{h,t+1}$, $b_{f,t+1}$, L_t and C_t, subject to (13) for all periods. As a result, Euler equations for share holdings can be derived as

$$1 = \beta E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} R_{h,t+1}^s \right],$$

Euler equations for bond holdings are given by

$$1 + \vartheta b_{h,t+1} = \beta E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} R_{h,t+1}^b \right], \quad 1 + \vartheta b_{f,t+1} = \beta E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} R_{f,t+1}^b \right].$$

Finally, the optimal labor supply is given by

$$\chi \left(L_t\right)^{\frac{1}{\psi}} = w_t C_t^{-\gamma}.$$

A.2 General Equilibrium and Net Foreign Asset Dynamics

We have the same labor market clearing condition as in the benchmark model. The balanced trade condition is, however, replaced by the following net foreign asset dynamics. Net foreign assets (in the Home consumption unit) at the end of period t are defined as

$$NFA_{t+1} \equiv b_{f,t+1}Q_t - b^*_{h,t+1}.$$

Since there are no cross-border equity holdings by assumption, only cross-border bond holdings appear in the definition. With the above definition of the net foreign assets, the budget constraint (13) can be rewritten and provides the following net foreign asset dynamics:

$$NFA_{t+1} = NX_t + NFA_t R^b_{h,t} + \xi_{h,t},$$

where NX_t denotes net exports and ξ_t stands for the "excess returns" between $t-1$ and t relative to returns on Home bonds $R^b_{h,t}$. Precisely, NX_t and ξ_t are given by

$$NX_t = \frac{1}{2}\left[w_t L_t + N_{D,t}\widetilde{d}_t - Q_t\left(w^*_t L^*_t + N^*_{D,t}\widetilde{d}^*_t\right)\right] - \frac{1}{2}\left[(C_t - N_{E,t}\widetilde{v}^s_t) - Q_t\left(C^*_t - N^*_{E,t}\widetilde{v}^{s*}_t\right)\right],$$

and

$$\xi_t \equiv b_{f,t}Q_t\left(R^b_{f,t} - R^b_{h,t}\right).$$

Note that the excess returns are zero in the first-order dynamics because of zero bond holdings due to adjustment costs in the steady state. Finally, asset markets clear for all time periods as

$$b_{h,t+1} + b^*_{h,t+1} = b_{f,t+1} + b^*_{f,t+1} = 0.$$

Table 6 summarizes the set of equations replaced or added. The symmetric steady state remains the same as in the model with balanced trade.

A.3 Sensitivity Analysis

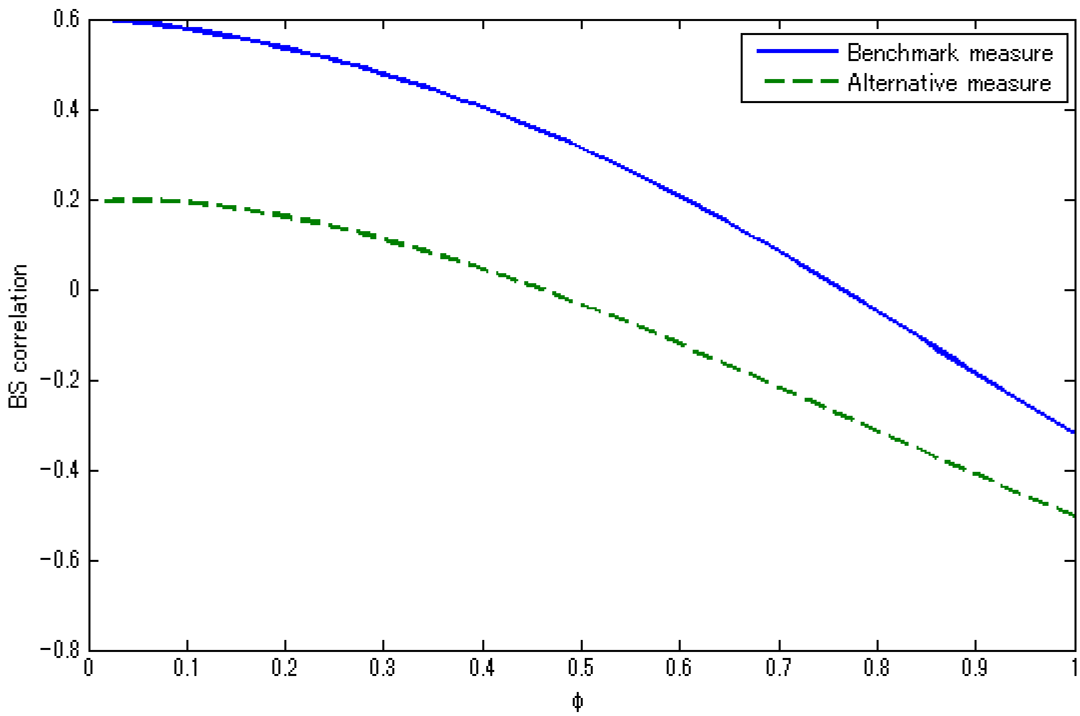
B Steady State

At the symmetric steady state, we assume without loss of generality that $Z = Z^* = f_E = f^*_E = z_{\min} = z^*_{\min} = 1$. In this symmetric steady state, we drop the asterisks, which

Table 6: The Mode with International Bonds

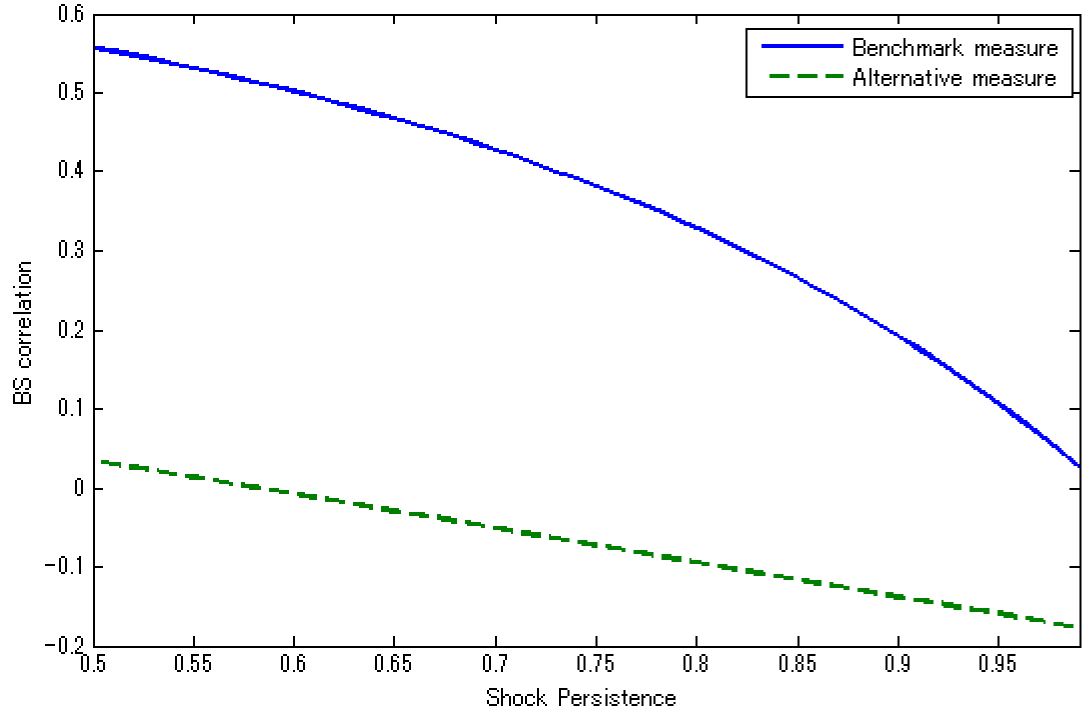
Euler bonds	$1+\vartheta b_{h,t+1}=\beta E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}R^b_{h,t+1}\right]$
	$1+\vartheta b_{f,t+1}=\beta E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}R^b_{f,t+1}\right]$
	$1+\vartheta b^*_{f,t+1}=\beta E_t\left[\left(\frac{C^*_{t+1}}{C^*_t}\right)^{-\gamma}R^b_{f,t+1}\frac{Q_t}{Q_{t+1}}\right]$
	$1+\vartheta b^*_{h,t+1}=\beta E_t\left[\left(\frac{C^*_{t+1}}{C^*_t}\right)^{-\gamma}R^b_{h,t+1}\frac{Q_t}{Q_{t+1}}\right]$
BMC	$b_{h,t+1}+b^*_{h,t+1}=0,\quad b_{f,t+1}+b^*_{f,t+1}=0.$
Net foreign Asset	$NFA_{t+1}=NX_t+NFA_t\left(1+r_{t+1}\right)+\xi_t$
Net export	$NX_t=\frac{1}{2}\left[w_tL_t+N_{D,t}\widetilde{d}_t-Q_t\left(w^*_tL^*_t+N^*_{D,t}\widetilde{d}^*_t\right)\right]$
	$-\frac{1}{2}\left[\left(C_t+N_{E,t}\widetilde{v}^s_t\right)-Q_t\left(C^*_t+N^*_{E,t}\widetilde{v}^{s*}_t\right)\right]$
Excess returns	$\xi_t=Q_tB_{*,t}\left(r^*_{t+1}-r_{t+1}\right)$

Figure 5: The Backus-Smith Correlation and Quality Ladder (Bond Economy)



Note: The figure reports the sensitivity result of the unconditional BS correlation in the theoretical model against the quality ladder, ϕ, with the benchmark measurement error ($\lambda_1=\lambda_2=\lambda_3=1.$) and the alternative measurement error ($\lambda_1=S_{ED}$, $\lambda_2=\lambda_3=1-S_{ED}$) obtained under the bond economy.

Figure 6: The Backus-Smith Correlation and Shock Persistence (Bond Economy)



Note: The figure reports the sensitivity result of the unconditional BS correlation in the theoretical model against the shock persistence of the productivity process, Z_t, with the benchmark measurement error ($\lambda_1 = \lambda_2 = \lambda_3 = 1.$) and the alternative measurement error ($\lambda_1 = S_{ED}$, $\lambda_2 = \lambda_3 = 1 - S_{ED}$) obtained under the bond economy.

denote Foreign variables and time indices. Note that $NFA = NX = 0$ and $Q = 1$ in the symmetric steady state. We choose the parameter χ so that the steady-state labor supply reaches unity as $L = 1$.

First, we solve the value of f_X so that it matches the empirical findings on the share of exporters. The free-entry condition gives $\widetilde{v}^s = w$. Thus, using the steady-state Euler equation for share holdings, we have

$$\widetilde{d} = \frac{1 - \beta(1-\delta)}{\beta(1-\delta)} w. \tag{14}$$

Therefore, by the definition of $\widetilde{d}$, we get

$$\widetilde{d}_D + \frac{N_X}{N_D}\widetilde{d}_X = \frac{1 - \beta(1-\delta)}{\beta(1-\delta)} w. \tag{15}$$

Now, we rewrite $\widetilde{d}_D$ and $\widetilde{d}_X$ in the above expression. From the zero-profit export cutoff condition, we have

$$\widetilde{d}_X = wf_X \frac{\sigma - 1}{k - (\sigma - 1)}. \tag{16}$$

With the above expression and using the steady-state average domestic and export profits $\widetilde{d}_D$ and $\widetilde{d}_X$, $\widetilde{d}_D$ can be rewritten as

$$\widetilde{d}_D = \frac{1}{\tau^{1-\omega}} \left(\frac{N_X}{N_D}\right)^{1-\psi(\omega-1)} \left(\frac{\widetilde{z}_X}{\widetilde{z}_D}\right)^{(1-\omega)(1+\phi)} \left[\frac{\sigma - 1}{k - (\sigma - 1)} + 1\right] wf_X, \tag{17}$$

where we use the fact that $\widetilde{\rho}_D/\widetilde{q}_D = \frac{\sigma}{\sigma-1}\frac{1}{1-\phi}\frac{w}{\widetilde{q}_D\widetilde{z}_D}$, $\widetilde{\rho}_X/\widetilde{q}_X = \frac{\sigma}{\sigma-1}\tau\frac{1}{1-\phi}\frac{w}{\widetilde{q}_X\widetilde{z}_X}$ and $\widetilde{q}_D = \left(\frac{\phi}{1-\phi}\widetilde{z}_D\right)^{\phi}$, $\widetilde{q}_X = \left(\frac{\phi}{1-\phi}\widetilde{z}_X\right)^{\phi}$.

Plugging (17) and (16) into (15), we get

$$\left[\frac{1}{\tau^{1-\omega}} \left(\frac{N_X}{N_D}\right)^{1-\psi(\omega-1)} \left(\frac{\widetilde{z}_X}{\widetilde{z}_D}\right)^{(1-\omega)(1+\phi)} \frac{k}{k - (\sigma - 1)} + \frac{N_X}{N_D}\frac{\sigma - 1}{k - (\sigma - 1)}\right] f_X = \frac{1 - \beta(1-\delta)}{\beta(1-\delta)}. \tag{18}$$

In the above expression, $\widetilde{z}_D$ is given by Pareto distribution. $\frac{N_X}{N_D}$ is set to 0.21. Given this value, which is also from the Pareto distribution, $\widetilde{z}_X = 2.9425$ is required with the values of parameters in the benchmark calibration. By plugging these values into the above equation, f_X can be solved.

Provided this subsidy, the steady-state labor supply is set to unity by controlling χ. Thus, the labor market clearing condition in the steady state gives

$$w = \left[N_E\widetilde{v}^s + (\sigma - 1) N_D\widetilde{d} + +\sigma N_X w f_X\right].$$

The equation about the motion of firms gives $N_E = \frac{\delta}{1-\delta}N_D$. Using (14) and replacing $\widetilde{v}^s$ as previously, the above expression can be rewritten as

$$N_D = \frac{1}{\frac{\delta}{1-\delta} + (\sigma - 1)\frac{1-\beta(1-\delta)}{\beta(1-\delta)} + \sigma\frac{N_X}{N_D}f_X}. \tag{19}$$

This is the solution for N_D.

Finally, the second equation can be obtained using the steady-state price index as

$$\left(\frac{\widetilde{z}_X}{\widetilde{z}_D}\right)^{(1-\omega)(1+\phi)} + \tau^{1-\omega}\left(\frac{N_X}{N_D}\right)^{-\psi(1-\omega)} = \left(\frac{N_D^{\psi}}{\frac{\sigma}{\sigma-1}\frac{1}{1-\phi}\frac{w}{\widetilde{q}_X\widetilde{z}_X}}\right)^{1-\omega} \tag{20}$$

By rearranging this equation, we have the solution for w:

$$w = \left\{ \left(N_D^{\psi} \frac{\sigma}{\sigma - 1} \frac{1}{1-\phi} \frac{1}{\widetilde{q}_X \widetilde{z}_X} \right)^{1-\omega} \left[\left(\frac{\widetilde{z}_X}{\widetilde{z}_D} \right)^{(1-\omega)(1+\phi)} + \tau^{1-\omega} \left(\frac{N_X}{N_D} \right)^{-\psi(1-\omega)} \right] \right\}^{\frac{1}{-(1-\omega)}}.$$

Once w is found, N_D can be found from (19). The steady-state value of other variables are relatively easy to find. In particular, the value of parameter χ is set by $\chi = wC^{-\gamma}$ so that $L = 1$. It gives 0.1829 with the parameter values of the benchmark calibration.

Finally, we define steady-state shares that appear in calibrating the first-order set of equations. The share of domestic and imported goods in total expenditures is

$$S_{ED} \equiv \rho_H^{1-\omega} \text{ and } 1 - S_{ED} \equiv \rho_F^{1-\omega}.$$

The steady-state share of fixed export costs, dividends on domestic, export and total sales relative to C are defined as

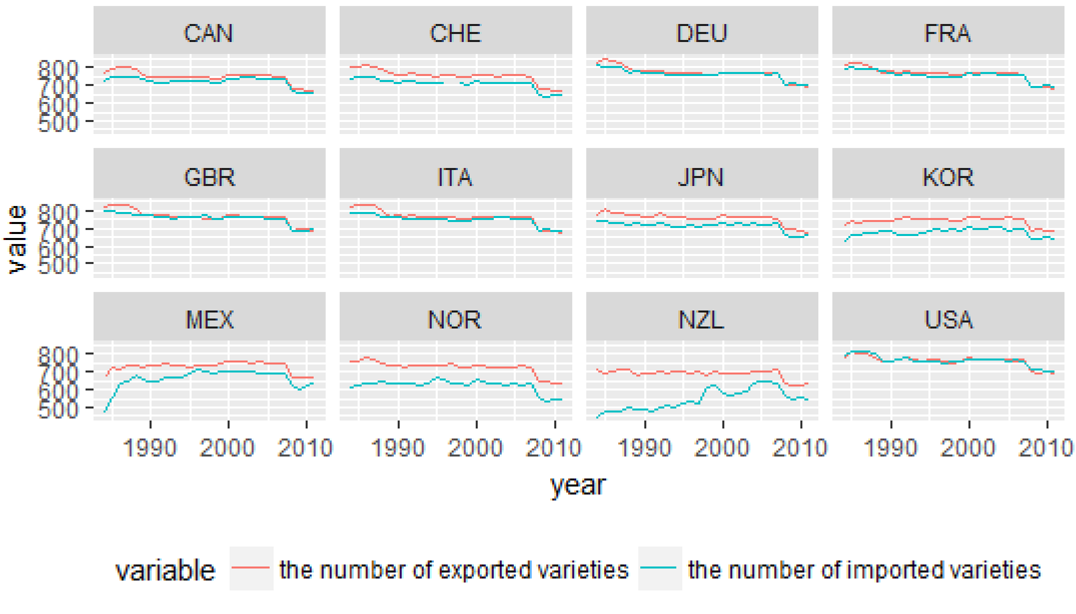
$$S_{FX} \equiv \frac{N_X w f_X}{C}, \quad S_{DD} \equiv \frac{N_D \widetilde{d}_D}{C}, \quad S_X \equiv \frac{N_X \widetilde{d}_X}{C}, \quad S_D \equiv \frac{N_D \widetilde{d}}{C}.$$

The steady state share of investments, wage and consumption relative to C are defined as

$$S_I \equiv \frac{N_E v^s}{C}, \quad S \equiv \frac{w}{C}, \quad S_M \equiv \frac{M}{C}.$$

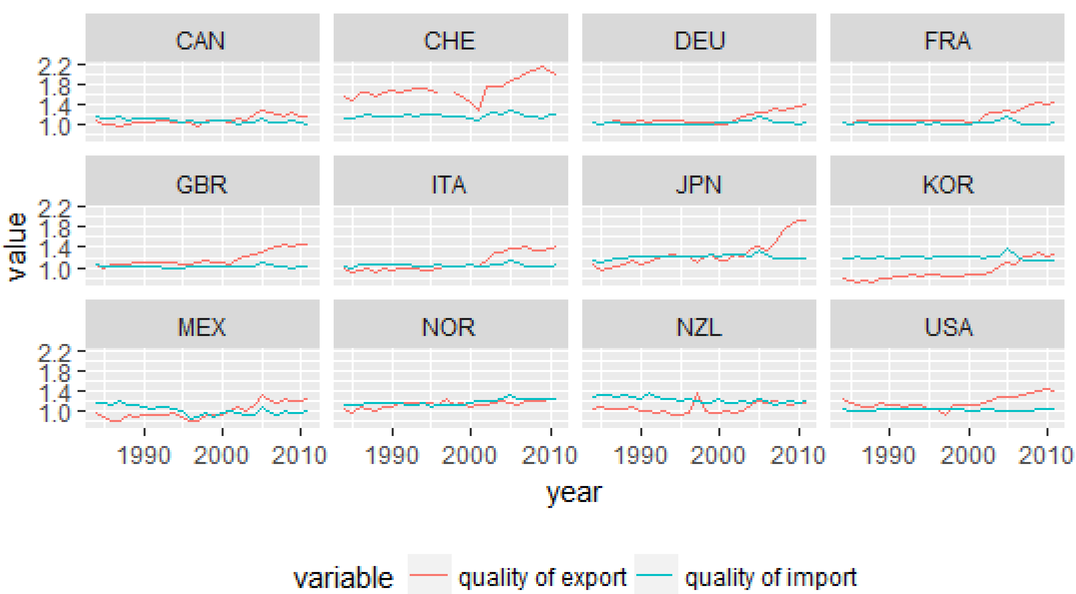
C Data

Figure 7: Evolution of the Number of Exported and Imported Varieties



Note: Evolution of the number of exported and imported varieties of the selected countries from 1984 to 2011. Source: Feenstra and Romalis (2014) and the author's calculation.

Figure 8: Evolution of Exported and Imported Quality



Note: Evolution of the number of exported and imported varieties of the selected countries from 1984 to 2011. Source: Feenstra and Romalis (2014) and the author's calculation.

Table 7: Descriptive Statistics

Statistic	N	Mean	St. Dev.	Min	Max
The number of exported varieties	713	740.0	37.7	623	845
Quality of exported goods	709	1.2	0.2	0.7	2.1
The number of imported varieties	709	696.1	60.4	441	816
Quality of imported goods	713	1.1	0.1	0.9	1.4

Source: Feenstra and Romalis (2014) and the author's calculation.