Imperfect Intersectoral Labor Substitutability and Welfare Cost of Business Cycles*

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Abstract

Positive implications of imperfect intersectoral labor substitutability on business cycles have been well-documented by previous literature, while studies on the normative implications of such a feature are rare. This paper fills this void by studying the implication of imperfect labor substitutability across sectors on the welfare cost of business cycles. With the neoclassical two-sector business cycle model that parsimoniously captures the intersectoral labor substitutability, we show that the estimated welfare cost from the model can be biased downward when the substitutability is assumed to be high. In particular, the uncertain environment can be welfare-improving when labor hours in different sectors are almost perfect substitutes to each other, which is an implicit assumption made in the one-sector model that has been typically used in the previous literature. A small departure from the assumption of perfect intersectoral labor substitutability, however, makes more volatile environment welfare-detrimental.

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1 Introduction

Since the seminal work by Lucas (1987), welfare cost of business cycles has become one of the main research topics in the field of macroeconomics. Among many others, see Krusell and Smith (1999); Storesletten, Telmer, and Yaron (2001); Mukoyama and Şahin (2006); Otrok (2001a); Otrok (2001b); Dolmas (1998); Cho, Cooley, and Kim (2015); and Lester, Pries, and Sims (2014) for example. One of the common underlying assumptions in this literature is that labor in different sectors are *perfectly* substitutes for each other. This is because they consider one-sector business cycle model that implicitly assumes $N_t = \sum_i N_{it}$ where N_{it} is the hours worked in sector i and N_t is the total hours worked supplied by households. One exception to use the two-sector model is Otrok (2001a) but this paper explicitly assumes perfect intersectoral labor substitutability.[1]

However, as is well-known in the literature on intersectoral labor substitutability, two important implications of the perfect substitutability assumption are at odds with the data. First, the theory predicts that the wage rates in different sectors should be the same with each other, at least from the perspective of workers, while there exist persistent interindustry wage differentials in the data (Krueger and Summers (1988)). Second, sectoral labor comovement problem arises under the perfect labor substitutability assumption. Figure 1.1 shows the dynamic relationship between GDP and employment of consumption goods producing sector (non-durable and services producing industries) and that of investment goods producing sector (construction and durable goods producing industries), whose data including quarterly real GDP (1947.I-2015.II) are downloaded from the FRED website. Here, we conduct simple bivariate VARs between GDP and sectoral employment to show that hours in different sectors comove in the data.[2] However, as is pointed out by previous literature including Boldrin, Christiano, and Fisher (2001), the one-sector business cycle model fails to reproduce such a pattern[3]; given the positive productiv-

[1] As a result, Otrok (2001a) did not take the utility from leisure into account when he computed the welfare cost of business cycles since the labor market is not well-explained with his model.

[2] See Christiano and Fitzgerald (1998) for further empirical evidence.

[3] For instance, Figure D.1, the collection of impulse response functions of hours in each sector to the technology shock when labor in different sectors are assumed to be perfect substitutes from our model, shows that hours in different sectors move in the opposite directions.

ity shock, employment increases in investment sector while it decreases in consumption sector since the demand for investment goods rises sharply. This problem also arises in the two-sector model that assumes labor in different sectors are perfect substitutes (see Otrok (2001a) as an example).

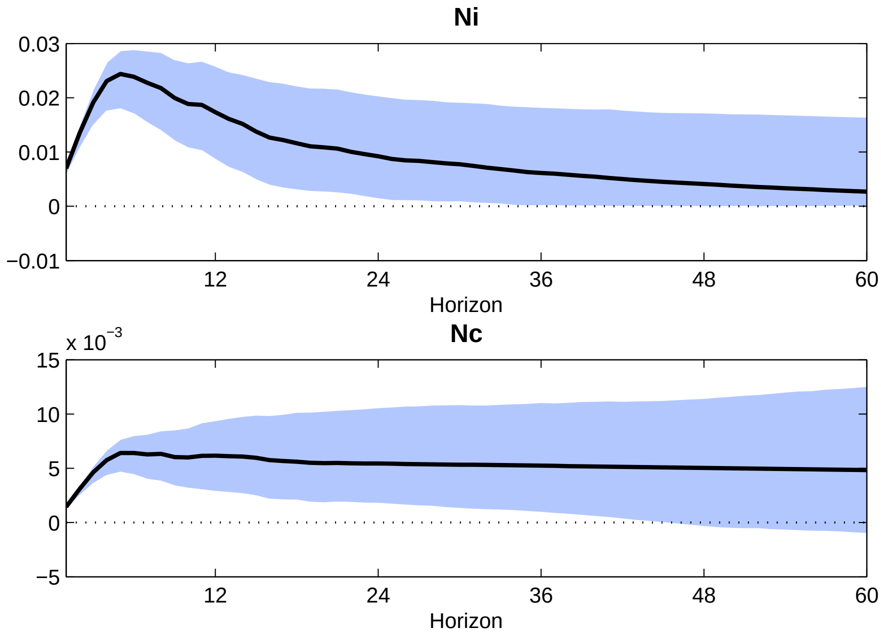

Figure 1.1: Sectoral Labor Comovement: Data
Note: Bias-corrected bootstrap confidence intervals, following Kilian (1998), are at 90% confidence level.

In order to deal with the above-mentioned problems, literature has allowed *imperfect* intersectoral labor substitutability. In particular, positive implications of such an assumption have been extensively studied by the previous literature. For instance, Boldrin, Christiano, and Fisher (2001) showed that the sectoral labor comovement problem can be resolved when the composition of labor is determined before the shock is realized. Huffman and Wynne (1999) showed that cross-sector behavior of employment observed in the data can be well reproduced with imperfect labor substitutability, assuming sector-specific technology shock. Katayama and Kim (2014) found that such a feature is important to obtain plausible business cycle fluctuations with news shocks. However, the study on normative implications of such a labor market feature on aggregate economy is rare. This paper fills this gap; we particularly study the extent to which varying the degree of intersectoral labor substitutability can affect the welfare cost of business cycles.

In so doing, we consider a neoclassical two-sector business cycle model that incorporates various features such as (1) habit formation, (2) non-separable preference between consumption and leisure, (3) intersectoral labor substitutability and intersectoral capital immobility, and (4) capital utilization. We find that imperfect intersectoral labor substitutability, which is one of the keys to obtain plausible sectoral labor market dynamics in the two-sector model, plays an important role in calculating the welfare cost. Most importantly, the estimated welfare cost from the model is decreasing in the degree of intersectoral labor substitutability, regardless of different parameter choices for labor supply elasticity, habit formation, and elasticity of intertemporal substitution. This implies that the welfare cost of business cycles computed by the previous literature with an one-sector model, which assumes extremely high labor substitutability, can be possibly biased downward.

The intuition is as follows. Suppose that the sector-neutral technology shock is the source of economic fluctuations. In the business cycle model without any obstacles to re-allocate resources between the sectors because the labor in different sectors are perfect substitutes as in the usual one-sector model, the consumer (and worker) can exploit the economic opportunity when the environment is volatile by utilizing resources as she wants; while the recession is bad for the consumer, she can instead enjoy higher utility when the economy is in the boom (mean effect; see Cho, Cooley, and Kim (2015) for details). Hence, as long as there is no constraint to alter the usage of resources, the consumer can enjoy the volatile environment as long as the mean effect is substantial. However, this economy fails to generate plausible labor market dynamics such as sectoral labor comovement. As the degree of labor substitutability becomes low, in contrast, it becomes more difficult for the consumer to re-allocate the resources across the sectors as she desires whenever the economy is hit by exogenous shocks. As a result, the welfare cost of business cycles would become greater in such an economy since the mean effect is lower.

Another innovation in our paper is methodological; we use the second-order approximation of equilibrium conditions, or perturbation methods, when computing the welfare cost of business cycles, as suggested by Schmitt-Grohé and Uribe (2004) and Schmitt-Grohé and Uribe (2006), which was not available when Otrok (2001a) was written. Interestingly, the welfare cost of business cycles computed from our baseline model is comparable to that from Otrok (2001a), which verifies the robustness of the previous results, starting from Lucas (1987), that the estimated welfare cost of business cycles are very low regardless of different models and different solution methods.

The rest of the paper is organized as follows. We first introduce a simplified two-sector model to get key intuitions in Section 2. Section 3 then introduces the main model. Then Section 4 presents our findings on the welfare cost of business cycles. Section 5 concludes.

2 A Simple Economy

In order to obtain key intuitions about the welfare cost of business cycles from the two-sector model, we introduce a simple model with two consumption goods, which is basically an extension of the simple model introduced in Lester, Pries, and Sims (2014). While we do not have investment good for analytical tractability, this model still provides the key insights of the general model. Further, the static environment is considered (1) for analytical characterization of the equilibrium and (2) because our interest lies in the contemporaneous relationship between hours supplied in different sectors. All the relevant proofs are provided in Appendix A.

The representative consumer's problem is given as follows.

$$\max_{c_{1t},c_{2t},N_{1t},N_{2t},N_t} \frac{c_{1t}^{1-\gamma}-1}{1-\gamma}+\frac{c_{2t}^{1-\gamma}-1}{1-\gamma}-\frac{N_t^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} \tag{2.1}$$

subject to

$$\begin{aligned}
(1)\ c_{1t} &= Z_{1t}N_{1t}\\
(2)\ c_{2t} &= Z_{2t}N_{2t}\\
(3)\ N_t &= \left[N_{1t}^{\frac{\theta+1}{\theta}}+N_{2t}^{\frac{\theta+1}{\theta}}\right]^{\frac{\theta}{\theta+1}}
\end{aligned}$$

where c_{it} is consumption good $i \in \{1,2\}$, N_t is the total labor, N_{it} is the labor input in sector i, γ is the risk aversion parameter, and η is the Frisch elasticity. Here, we introduce sector-specific technology shock, Z_{it}, instead of introducing general shock Z_t. One can show that in this class of simple models with two consumption goods, sector-neutral technology shock cannot generate both wage differentials across sectors and sectoral labor comovement problem while sector-specific shock can. This is not limited to the assumption of linear technology; this holds even when the production function exhibits decreasing return to scale in labor ($c_{it} = Z_t N_t^{\alpha}$ where $\alpha \in (0,1)$ for instance).

The last constraint (constraint (3)) captures the idea that it incurs some costs when reallocating market hours from one sector to other sector, following Huffman and Wynne (1999) and Horvath (2000). In particular, the elasticity of intratemporal substitution, θ, which controls the degree to which labor can move across sectors, satisfies the following equation:

$$\frac{d\ln\left(\frac{N_{1t}}{N_{2t}}\right)}{d\ln\left(\frac{w_{1t}}{w_{2t}}\right)} = \theta \tag{2.2}$$

where the above condition comes from the labor supply equation.

θ measures the extent to which the relative labor in different sectors respond to the relative wage. If $\theta \to \infty$, all sectors should pay the same hourly wage. If not, only the high-wage sector will hire the workers. Hence, labor hours devoted in different sectors are perfect substitutes in this case. One can interpret the usual one-sector model as the nested version of our model when $\theta \to \infty$ (together with more assumptions to make capital moves freely across sectors). In contrast, when $\theta \to 0$, it is impossible to change the composition of labor hours between the sectors since relative hours do not respond to changes in relative wages. i.e. it incurs infinite costs to move labor from one sector to other sector. This is the limiting case equivalent to the setup employed by Boldrin, Christiano, and Fisher (2001); they assumed that labor decision is made before the shock hits the economy. i.e. the composition of labor devoted in each sector is fixed at the realization of the shock. In the intermediate case, $0 < \theta < \infty$, the worker diversifies her labor hours by allocating positive hours in each sector. In this case, wages in different sectors can be divergent. Hence, the labor market setup employed in our paper is more flexible and convenient than the previous papers in the sense that we can simply change the value of the parameter θ to alter the degree of intersectoral labor substitutability. This environment also captures the idea that there can be a sector-specific human capital so that one cannot easily change the composition of labor in different sectors (it incurs some cost).

Solution for labor inputs are derived as follows.

$$N_{1t} = \left(\kappa\left(\frac{Z_{1t}}{Z_{2t}}\right)^{\frac{\theta+1}{\theta}} - 1\right)^{\frac{\theta}{\theta+1}} \kappa\left(\frac{Z_{1t}}{Z_{2t}}\right)^{\frac{\frac{1}{\theta}-\frac{1}{\eta}}{\frac{1}{\eta}+\gamma}} Z_{2t}^{\frac{1-\gamma}{\frac{1}{\eta}+\gamma}} \tag{2.3}$$

$$N_{2t} = \kappa\left(\frac{Z_{1t}}{Z_{2t}}\right)^{\frac{\frac{1}{\theta}-\frac{1}{\eta}}{\frac{1}{\eta}+\gamma}} Z_{2t}^{\frac{1-\gamma}{\frac{1}{\eta}+\gamma}} \tag{2.4}$$

$$N_t = \kappa\left(\frac{Z_{1t}}{Z_{2t}}\right)^{\frac{\gamma+\frac{1}{\theta}}{\frac{1}{\eta}+\gamma}} Z_{2t}^{\frac{1-\gamma}{\frac{1}{\eta}+\gamma}} \tag{2.5}$$

where $\kappa\left(\frac{Z_{1t}}{Z_{2t}}\right) \equiv \left[\left(\frac{Z_{1t}}{Z_{2t}}\right)^{\frac{(1+\theta)(1-\gamma)}{\gamma\theta+1}} + 1\right]^{\frac{\theta}{\theta+1}}$. We note that $\kappa(\cdot)$ is increasing in Z_{1t} for given Z_{2t} when $0 < \gamma < 1$ but is decreasing in Z_{1t} when $\gamma > 1$. C_{it} then automatically follow.

Before studying the welfare cost of business cycles with the simple model, we note that the closed-form solution we obtain above is not a linear function of productivity variables. As a result, the value function of the consumer is a non-trivial function of Z_{1t} and Z_{2t}, which makes the analytic analysis of the welfare cost from changes in Z_{1t} difficult. Hence we instead consider two extreme cases to keep analytical tractability; (1) $\theta \to 0$ (perfect intersectoral labor complementarity) and (2) $\theta \to \infty$ (perfect intersectoral labor substitutability).[4]

Case 1: Perfect intersectoral labor complementarity. We first consider the economy where the composition of labor hours is fixed (equivalent to Boldrin, Christiano, and Fisher (2001); in their model, the composition is fixed only at the first period). The solution becomes simple as described in the following Lemma.

Lemma 1 (Solution when $\theta \to 0$: fixed composition of labor hours)**.** *Suppose that $\theta \to 0$. Then*

[4]We also study the condition for the sectoral labor comovement in Appendix A.1.

$$N_{1t} = N_{2t} = N_t = Z_{2t} \tag{2.6}$$

Proof. First, we can show that $\kappa(\cdot)$ approaches one as θ approaches zero. Then N_{2t} converges to Z_{2t}. Then equation (2.3) implies that N_{1t} also approaches Z_{2t}, given $0^0 = 1$. Then $N_t = Z_{2t}$ automatically follows. □

The solution is intuitive; when θ becomes zero labor hours in each sector are perfect complement to each other hence the ratio should be constant. Then it follows that $c_{1t} = Z_{1t}Z_{2t}$ and $c_{2t} = Z_{2t}^2$. Suppose that Z_{1t} increases. The consumer would like to put more hours in sector 1 to exploit this good business opportunity but the fixed composition of labor prohibits this change. Hence, there only is an income effect on c_{1t} from changes in Z_{1t} without changes in labor.

Then how does the changes in Z_{1t}, which changes the wage paid in sectors asymmetrically, affect the welfare cost of business cycles? Using the utility function of the consumer, we can obtain the value of the consumer as follows.

$$V(Z_{1t}) = \frac{(Z_{1t}Z_{2t})^{1-\gamma} - 1}{1-\gamma} + \frac{Z_{2t}^{2(1-\gamma)} - 1}{1-\gamma} - \frac{Z_{2t}^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} \tag{2.7}$$

Then

$$V'(Z_{1t}) = Z_{1t}^{-\gamma} Z_{2t}^{1-\gamma} > 0 \tag{2.8}$$

and

$$V''(Z_{1t}) = -\gamma Z_{1t}^{-\gamma-1} Z_{2t}^{1-\gamma} < 0 \tag{2.9}$$

It is straightforward that value of the consumer is concave in Z_{1t}. As a result, Jensen's inequality implies that mean-preserving spread of Z_{1t} is not preferred by the consumer.

Proposition 1 (Welfare cost with perfect intersectoral labor complementarity)**.** *Suppose that* $\gamma \in (0, 1)$ *and* $\theta \to 0$. *Assume, in addition, that the source of the fluctuation comes from the shock to* Z_{1t}. *Then the welfare cost of business cycles is always positive regardless of the combination* *between γ, risk aversion parameter, and η, the Frisch elasticity. Furthermore, there is no sectoral labor comovement problem in this economy.*

Hence, in the economy where $\theta \to 0$ so that there is no sectoral labor comovement problem (i.e. model's prediction is consistent with the observed patterns in data), the positive shock to Z_{1t} always decreases the welfare regardless of the combination of γ and η. This is a sharp contradiction to what Lester, Pries, and Sims (2014) found with an one-sector model. In their simple model, for given some γ, the consumers can prefer more volatile environment when the Frisch elasticity (η) is sufficiently high, which is not the case in our model with low intersectoral labor substitutability. This comes from the fact that consumers cannot change the composition of hours when the shock to Z_{1t} hits the economy so that the positive effect of business cycles is minimized.

Case 2: Perfect intersectoral labor substitutability. Suppose now that $\theta \to \infty$. Hence, labor are perfect substitutes across different sectors. To keep the analysis simple as possible, we normalize Z_{2t} as 1. Then

Lemma 2 (Solution when $\theta \to \infty$: labor hours in different sectors are perfect substitutes)**.** *Suppose that $\theta \to \infty$ and we normalize Z_{2t} as one. Then*

$$N_{1t} = Z_{1t}^{\frac{1-\gamma}{\gamma}} N_{2t} \quad N_{2t} = \left(Z_{1t}^{\frac{1-\gamma}{\gamma}} + 1 \right)^{-\frac{\frac{1}{\eta}}{\frac{1}{\eta}+\gamma}} \quad N_t = N_{2t}^{-\gamma\eta} \tag{2.10}$$

Then it is easy to check that $c_{1t} = Z_{1t}^{\frac{1}{\gamma}} N_{2t}$ and $c_{2t} = N_{2t}$ and can show that the sectoral comovement problem arises in this case.[5] Now the value function of the consumer is

$$V(Z_{1t}) = \frac{\frac{1}{\eta} + \gamma}{(1-\gamma)(1+\frac{1}{\eta})} \left(Z_{1t}^{\frac{1-\gamma}{\gamma}} + 1 \right)^{\frac{\gamma(1+\frac{1}{\eta})}{\frac{1}{\eta}+\gamma}} \tag{2.11}$$

Differentiating the value function with respect to Z_{1t}, we obtain:

[5] See Proposition 3 in Appendix A.1.

$$V'(Z_{1t}) = \left(Z_{1t}^{\frac{1-\gamma}{\gamma}} + 1\right)^{\frac{\gamma(1+\frac{1}{\eta})}{\frac{1}{\eta}+\gamma}-1} Z_{1t}^{\frac{1}{\gamma}-2} > 0 \tag{2.12}$$

Differentiating again, we obtain

$$V''(Z_{1t}) \propto \left(-\frac{\frac{(1-\gamma)^2}{\eta}}{\frac{1}{\eta}+\gamma} + \frac{1}{\gamma} - 2\right) Z_{1t}^{\frac{2}{\gamma}-4} + \left(\frac{1}{\gamma} - 2\right) Z_{1t}^{\frac{1}{\gamma}-3} \tag{2.13}$$

Hence, the welfare cost of business cycles in this economy can be described as follows.

Proposition 2 (Welfare cost with perfect intersectoral labor substitutability)**.** *Suppose that* $\gamma \in (0, \frac{1}{2})$ *and* $\theta \to \infty$*. Assume, in addition, that the source of the fluctuation comes from the shock to* Z_{1t}*. Then the consumers prefer volatile environment if and only if* $\left(Z_{1t}^{1-\frac{1}{\gamma}} + 1\right)\left(\frac{1}{\gamma} - 2\right) > \frac{(1-\gamma)^2}{1+\eta\gamma}$*. Furthermore, the sectoral comovement problem arises in this economy.*

For instance, suppose that $Z_{1t} = 1$ and $\eta \to \infty$. i.e. labor supply is infinitely elastic. Then, contrary to the economy with fixed composition of labor, the welfare cost of business cycles can be negative for some parameter regions. We further note that $\frac{(1-\gamma)^2}{1+\eta\gamma}$ is decreasing in η. This implies that for given γ, the condition for the above lemma to hold is easier to achieve when the Frisch elasticity is high, which is consistent with the predictions of Cho, Cooley, and Kim (2015) and Lester, Pries, and Sims (2014).

In summary, our analytical exercise clearly shows the importance of considering the degree of intersectoral labor substitutability when computing the welfare cost of business cycles. With low intersectoral labor substitutability, the consumer cannot fully enjoy the more volatile environment hence the welfare cost of business cycle becomes always positive. With high intersectoral labor substitutability, in contrast, more volatile environment can be preferred by the consumer.

3 The Model

The main model is taken from Katayama and Kim (2014); while the benchmark model introduced here includes various shocks, including preference shock, aggregate neutral technology (TFP) shock, sector-specific technology shock, and investment-specific shock, the main results presented in Section 4 are obtained under the assumption that there is only one aggregate shock, aggregate TFP shock, to make our analysis consistent with the previous literature. Equilibrium conditions are summarized in Appendix B.

3.1 The Setup

The economy consists of identical households and firms.

Households. The economy is populated by a constant number of identical and infinitely-lived households. The representative household receives utility from the consumption and incurs disutility from allocating labor hours to the consumption and investment goods sectors. Let C_t and N_t respectively denote period t consumption and an aggregate labor index. Households maximize expected lifetime utility as given by

$$U_0 = E_0 \left[\sum_{t=0}^{\infty} \beta^t b_t U(C_t - hC_{t-1}, N_t) \right] \tag{3.1}$$

where $\beta \in (0,1)$ is the subjective discount factor and b_t is discount factor shock. The discount factor shock follows

$$b_t = (1-\rho_b)b + \rho_b b_{t-1} + \xi_{b,t}. \tag{3.2}$$

where b is the steady-state value. We quickly note that the exogenous shock, including the shocks introduced below, is specified in levels, following Lester, Pries, and Sims (2014). If the shock is specified in log, it suffers from the problem that the mean level of b_t increases when the variance of innovation increases.

The specific form of U adopted in this paper is as follows, which nests King-Plosser-Rebelo (KPR, hereafter) preference (King, Plosser, and Rebelo (1988)):

$$U(C_t - hC_{t-1}, N_t) = \frac{(C_t - hC_{t-1})^{1-\frac{1}{\sigma}} \left(1 + \left(\frac{1}{\sigma} - 1\right) v(N_t)\right)^{\frac{1}{\sigma}} - 1}{1 - \frac{1}{\sigma}} \tag{3.3}$$

where $v(N_t) = \nu \frac{\eta}{1+\eta} N_t^{\frac{\eta+1}{\eta}}$. $v(N_t)$ measures the disutility incurred from hours worked with $v^{'} > 0$, $v'' > 0$ and η is a Frisch elasticity of aggregate labor supply when preference is separable.

The representative household is assumed to be endowed with one unit of time in each period and the aggregate labor index N_t takes the following form:

$$N_t = \left[N_{c,t}^{\frac{\theta+1}{\theta}} + N_{i,t}^{\frac{\theta+1}{\theta}} \right]^{\frac{\theta}{\theta+1}}, \quad \theta \geq 0 \tag{3.4}$$

Here N_t is an aggregate labor hours index, and $N_{c,t}$ and $N_{i,t}$ respectively denotes labor hours devoted to the consumption and investment sector.

One issue raised in using non-separable preferences is to check the conditions for the overall concavity of momentary utility function and the normality of consumption and leisure, as emphasized by Bilbiie (2009). As long as $\sigma \leq 1$, the overall concavity of $U(\cdot)$ is guaranteed. To ensure the normality of consumption and leisure, we impose the following restriction to $U(\cdot)$:

$$-\left(N\frac{U_{LL}}{U_L} - N\frac{U_{CL}}{U_C} \right) = -\omega_N + \frac{v''(N)N}{v'(N)} > 0 \tag{3.5}$$

where $\omega_N \equiv \frac{\left(\frac{1}{\sigma}-1\right)v'(N)N}{\left(1+\left(\frac{1}{\sigma}-1\right)v(N)\right)}$ and $\lim_{\sigma\to 1}\omega_N = 0$.

Maximization occurs subject to the budget constraint

$$C_t + \left(\frac{P_{i,t}}{P_{c,t}}\right)(I_{c,t} + I_{i,t}) \leq \sum_{j=c,i}\left(\frac{W_{j,t}}{P_{c,t}}\right)N_{j,t} + \sum_{j=c,i}\left(\frac{R_{j,t}}{P_{c,t}}\right)u_{j,t}K_{j,t} \tag{3.6}$$

where the subscript c and i denote variables that are specific to the consumption and investment sector, respectively. $P_{j,t}$ is the nominal prices in sector $j = c, i$, $I_{j,t}$ represents newly purchased capital in sector j, and $W_{j,t}$ is the nominal wage rate paid by firms in sector j. In addition, $K_{j,t}$ is a productive capital stock in sector j and $u_{j,t}$ denotes the capital utilization rate in sector j. Hence, $u_{j,t}K_{j,t}$ represents the capital services and $R_{j,t}$ is the rental rates of capital services in sector j.

The capital stock in each sector $j = c, i$ evolves according to

$$K_{j,t+1} = I_{j,t}\left[1 - \phi\left(\frac{I_{j,t}}{I_{j,t-1}}\right)\right] + [1 - \delta(u_{j,t})]K_{j,t}, \quad j = c, i \tag{3.7}$$

Here $\phi(\cdot)$ represents adjustment costs that are occurred when the level of investment changes over time. We assume that ϕ_K and $\phi_K^{'} = 0$, and $\phi_K^{''} > 0$ in steady state. The function $\delta(\cdot)$ represents the depreciation rate. We assume that depreciation is convex in the rate of utilization: $\delta^{'} > 0$, $\delta^{''} \geq 0$. The function form of $\phi(\cdot)$ and $\delta(u_{j,t})$ is given by

$$\delta(u_{j,t}) = \delta_j^0 + \delta_j^1(u_{j,t} - 1) + \frac{\delta_j^2}{2}(u_{j,t} - 1)^2 \tag{3.8}$$

$$\phi\left(\frac{I_{j,t}}{I_{j,t-1}}\right) = \frac{\kappa_j}{2}\left(\frac{I_{j,t}}{I_{j,t-1}} - G^C G^V\right)^2 \tag{3.9}$$

where $G^C G^V$ denotes the steady-state growth rate of investment.

Firms. Output in each sector is produced by perfectly competitive firms with the Cobb-Douglas production function

$$C_t = Z_t z_{c,t}(u_{c,t}K_{c,t})^{\alpha}(N_{c,t})^{1-\alpha} \tag{3.10}$$

$$I_t = I_{c,t} + I_{i,t} = Z_t Z_{i,t}(u_{i,t}K_{i,t})^{\alpha}(N_{i,t})^{1-\alpha} \tag{3.11}$$

where Z_t is an aggregate total factor productivity (TFP) shock. $z_{c,t}$ is a sectoral TFP shock in the consumption sector and $Z_{i,t}$ is a sectoral TFP shock in the investment sector. Z_t has the two components:

$$Z_t = A_t a_t \tag{3.12}$$

where A_t denotes non-stationary shock and a_t denotes stationary shock. The growth rate of the permanent aggregate TFP shock, $z_t \equiv \log\left(\frac{A_t}{A_{t-1}}\right)$ follows AR(1) process:

$$z_t = (1 - \rho_z)z + \rho_z z_{t-1} + \xi_{z,t} \tag{3.13}$$

where z is the growth rate of permanent aggregate TFP A_t in steady state, and $\xi_{z,t}$ is i.i.d disturbance and represents a conventional TFP growth shock.

$Z_{i,t}$ has the two components:

$$Z_{i,t} = V_t z_{i,t} \tag{3.14}$$

where V_t denotes non-stationary investment-specific technology shock and $z_{i,t}$ denotes stationary investment-specific technology shock. The growth rate of the permanent investment-specific TFP shock, $v_t \equiv \log\left(\frac{V_t}{V_{t-1}}\right)$ follows AR(1) process:

$$v_t = (1-\rho_v)v + \rho_v v_{t-1} + \epsilon_{v,t-p} + \xi_{v,t} \tag{3.15}$$

where v is the growth rate of permanent investment-specific TFP V_t in steady state, and $\xi_{v,t}$ is i.i.d disturbances and represents a conventional investment-specific TFP growth shock.

We assume that the stationary aggregate TFP shock, a_t follow a mean zero AR(1) process in log.

$$a_t = (1-\rho_a)a + \rho_a a_{t-1} + \xi_{a,t} \tag{3.16}$$

where $\xi_{a,t}$ is i.i.d disturbance and represents a conventional contemporaneous stationary TFP shock.

We also assume stationary sectoral TFP shocks, which follow a mean zero AR(1) process in log.

$$z_{j,t} = \rho_j z_{j,t-1} + \xi_{j,t}, \quad j = c, i \tag{3.17}$$

where $z_{j,t}$ is sectoral TFP in sector $j = c, i$, and $\xi_{j,t}$ is i.i.d disturbance and represents a conventional sectoral TFP shock.

3.2 Computation of Welfare Cost of Business Cycles

We compare the value of lifetime utility drawn from non-fluctuating variables at the steady-state and that drawn from fluctuating variables around the steady-state. Formally, the value of the non-fluctuating economy is given by

$$V^{NF} = \sum_{t=0}^{\infty} \beta^t U\left((1-h)C, N\right) \tag{3.18}$$

where variables without subscript are steady-state variables and NF denotes non-fluctuating economy. Here, we assume that there is no trend for expositional simplicity. i.e. A_t and V_t are constant.

Then the value of the fluctuating economy would be

$$V^{F,\lambda} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U\left((1+\lambda)(C_t - hC_{t-1}), N_t\right) \tag{3.19}$$

and

$$V^{F} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t - hC_{t-1}, N_t\right) \tag{3.20}$$

Hence, λ is the compensating variation as usually used in the literature; it measures the percentage by which average consumption has to be increased for the consumer to be indifferent between the certain path of consumption and the volatile one. i.e. λ is the solution of the following equation:

$$V^{NF} = V^{F,\lambda} \tag{3.21}$$

Using the utility specification introduced in (3.3), we can solve for λ as follows.

$$\lambda = \left(\frac{V^{NF}}{V^{F}}\right)^{\frac{\sigma}{\sigma-1}} - 1 \tag{3.22}$$

In this paper, we follow Schmitt-Grohé and Uribe (2006) and Lester, Pries, and Sims (2014) when computing welfare cost of business cycles; we compute the conditional welfare cost of business cycles in this economy whose initial steady-state is $x_0 = x$ and the scaling parameter, σ_z, is set to be 0 as follows.

$$\lambda(x,0) = \left[\frac{V'(N)N}{\sigma\left(1+\left(\frac{1}{\sigma}-1\right)V(N)\right)} n_{\sigma_z\sigma_z} - c_{\sigma_z\sigma_z} \right] \frac{\sigma_z^2}{2} \tag{3.23}$$

where the derivation of the above formula is in the Appendix C.

4 Welfare Cost of Business Cycles

In this section, we present our main findings from the quantitative exercises.

4.1 Parameterization In the benchmark model, we use the parameters that are estimated in Katayama and Kim (2014). Table 4.1 summarizes the parameter values for the benchmark case. Without loss of generality, we assume that there is no exogenous growth in the model.

Table 4.1: Calibration

Parameter	Value	Description
α	0.36	Capital share
β	0.985	Discount factor
δ^0	0.025	Steady-state depreciation rate
δ^1	0.0402	Coefficient in depreciation function
δ^2	0.025	Coefficient in depreciation function
κ_C	4.4186	Coefficient in capital adjustment cost in consumption sector
κ_I	1.0939	Coefficient in capital adjustment cost in investment sector
h	0.2305	Habit formation
η	1.25	Frisch elasticity
θ	0.3014	Degree of intersectoral labor substitutability
σ	0.8392	Elasticity of intertemporal substitution
ρ_a	0.9592	AR (1) coefficient of stationary TFP shock
σ_a	0.01	s.d. of stationary TFP shock (normalization)

Note: δ^i where $i = 1, 2, 3$ are the same between the two sectors hence we abstract from the subscript $j = C, I$.

In our benchmark economy, we calibrate θ to be smaller than one. Since it is the parameter of main interest in this paper, we will discuss more about θ later. σ is about 0.8, hence the degree of non-separability in the utility function takes moderate value. η, Frisch elasticity when the utility function is separable, is about 1.25. h, the parameter that governs the habit formation, is calibrated as 0.23. This can be roughly compared to the habit formation parameter in Otrok (2001a), which is estimated to be about 0.446; hence h is smaller than what Otrok (2001a) and other earlier studies that used DSGE-based Bayesian estimation technique estimated. However, the low h is comparable to the estimates from the micro level studies such as Dynan (2000).

In Figure D.2, we show that our model can generate the plausible sectoral labor comovement under the above parameterization while assuming extremely high θ is not able to reproduce such a pattern (Figure D.1).

4.2 Main Results

This subsection provides several key results on the welfare cost of business cycles drawn from our model.

Welfare Cost: Benchmark Model. With our preferred parameterization, we solve the model with the second-order approximation of the equilibrium conditions (perturbation methods), which was originally suggested by Schmitt-Grohé and Uribe (2004). Then we compute the conditional welfare cost of business cycles by applying equation (3.23). This is the first contribution of our paper, which is a methodological innovation. To our best knowledge, we are the first to apply the perturbation methods in the two-sector model in computing the welfare cost of business cycles. For instance, Otrok (2001a) used the first-order approximation method (log-linearization) to find the equilibrium. Lester, Pries, and Sims (2014) introduced the investment-specific shock but they used the one-sector model instead of two-sector model. Our two-sector model nests their model as a limiting case where there is no factor immobility.

Table 4.2: Welfare Cost of Business Cycles: a Benchmark Case

Parameter	Welfare Cost (%)
Benchmark Case	0.00303

Two observations are noteworthy here. First, as found by many other previous studies, which was first emphasized by Lucas (1987), the welfare gain from eliminating business cycles is negligibly low. Moreover, the welfare cost we obtain from our benchmark model is comparable to that obtained by Otrok (2001a) while the models used in both papers are different; with his baseline model, he obtained 0.0044% as the welfare cost, which is also very low. We demonstrate why our welfare cost estimate is lower than Otrok (2001a)'s estimate below.

Second, while the welfare cost is low, it is definitely positive; this is an interesting point since Cho, Cooley, and Kim (2015) and Lester, Pries, and Sims (2014) find that welfare cost of business cycles can be negative under plausible parameter values in the variant versions of modern business cycle model. Contrary to their argument, however, our preferred calibration shows that it is desirable to eliminate economic fluctuations. Figure 4.1 presents the main result from our model; we compute the welfare costs of business cycles for different values of θ while keeping other parameters fixed. It is easy to observe that welfare cost of business cycles is decreasing in θ; as the degree of intersectoral labor substitutability is high (high θ), the estimated welfare cost becomes low. In contrast, with substantially low θ so that the intersectoral labor substitutability is low, welfare cost of business cycles is relatively high. Furthermore, it requires θ larger than 100 to obtain negative welfare costs. This implies that the welfare cost obtained from the one-sector model, which assumes extremely high θ, can be potentially biased downward.

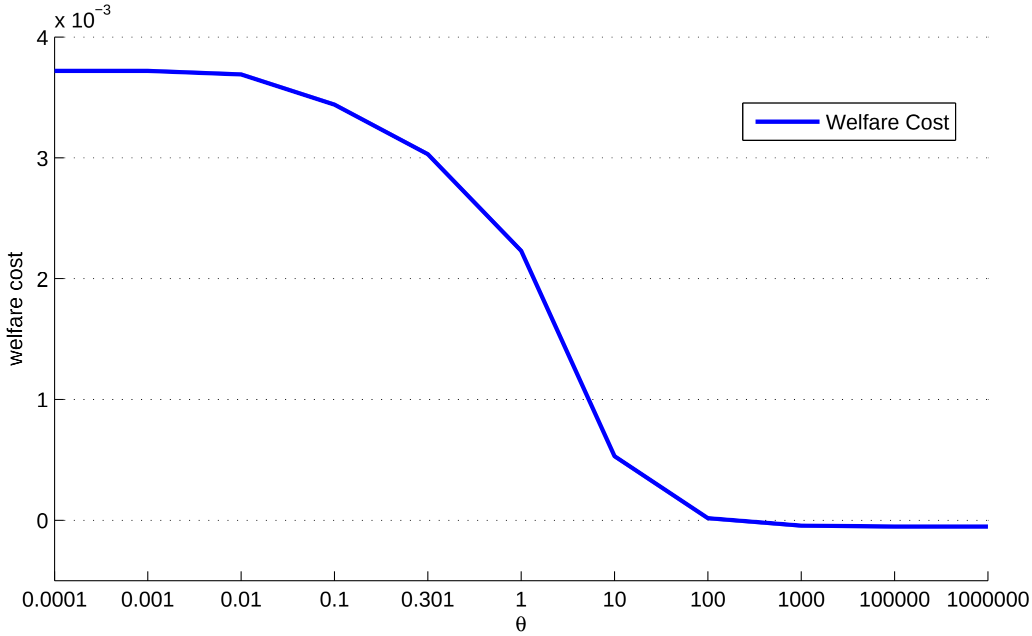

Figure 4.1: Welfare Cost (%): varying degree of intersectoral labor substitutability

We also note that in order for obtaining the plausible sectoral labor comovement, our model requires $\theta \leq 1$ when other parameters are assumed to be fixed as in Table 4.1. For instance, Horvath (2000) estimated θ as 1 instead of 0.30 estimated by Katayama and Kim (2014), but both values are able to resolve the sectoral labor comovement and imply positive welfare costs. Hence, we can infer from Figure 4.1 that it is more likely that the welfare cost of business cycles is positive when the business cycle model takes the sectoral labor comovement into account.

Welfare Cost: Robust to Different Parameter Values? In order to check if the positive welfare cost of the business cycles we obtain from the baseline model is robust, we compute the welfare cost by varying key parameters in the utility function, h, σ, and η. Table 4.3 summarizes the results.

Table 4.3: Welfare Cost of Business Cycles: Robustness to Alternative Parameter Values

h	0	0.2	0.4	0.6	0.8	0.95
Welfare Cost (%)	0.0037253	0.0031223	0.0025406	0.0022583	0.0026641	0.0045497
σ	0.1	0.3	0.5	0.7	0.9	0.99
Welfare Cost (%)	0.00091114	0.001533	0.0021646	0.0027068	0.0031522	0.0033248
η	0.2	0.25	0.33	0.5	1	2
Welfare Cost (%)	0.0030909	0.0030813	0.0030681	0.00304888	0.0030163	0.0029867

Note: In each experiment, we fix other parameter values to the benchmark case.

We quickly summarize key observations. First, the effect of changes in the habit formation, i.e. different choice of h, is not substantial and has a non-linear relationship with the welfare cost of business cycles. σ, the parameter that governs the substitutability between consumption and leisure, is positively related to the welfare cost of business cycles. In other words, as the substitutability between consumption and leisure becomes higher (lower σ), the welfare cost becomes lower. Hence, the welfare cost of business cycle obtained from the conventional business cycle model where consumption and leisure are separably additive in the utility function can be understood as the upper bound in this regard. Lastly, increases in η lowers the welfare cost of business cycles, which is consistent with Cho, Cooley, and Kim (2015) and Lester, Pries, and Sims (2014). This comes from the fact that the workers can exploit the opportunities from the economic fluctuations when the labor supply is relatively flexible. One major difference from their papers is that the welfare cost is always positive with different η; according the two papers, the welfare cost can be negative when the factor supply is sufficiently elastic. However, while higher η lowers the gains from removing the fluctuations, it does not yield the negative welfare cost in our model.[6] This comes from the fact that the worker cannot freely move her labor hours from one sector to other sector as she desires when the productivity shock hits the economy, which is already discussed in the earlier section.

Overall, our finding that our two-sector model, which is more consistent with observed labor market dynamics than the usual model considered in the previous literature, yields the positive welfare cost is robust to different set of parameter values. While the welfare cost is substantially low in any cases, which is not of our model's own problem, our experiments show the robustness of our finding that eliminating the business cycles would be beneficial for the consumers in this economy.

Welfare Cost: Role of Capital Immobilities. In our model, there are two inputs, capital and labor. Hence, it is natural to study the role of sectoral capital immobility. In this section, we study the possible interactions between sectoral capital immobility and sectoral labor substituability in computing welfare cost of business cycles. In doing so, we first note that there are three important parameters in this exercise; θ, κ, and δ^2.

In particular, we study the welfare cost of business cycles under the following three scenarios:

- Economy 1: Economy with no convex utilitization cost ($\delta^2 = 0$)
- Economy 2: Baseline economy ($\delta^2 = 0.025$)
- Economy 3: Economy with high convex utilitization cost ($\delta^2 = 1,000,000$)

The first economy exhibits the property that physical capital is perfectly mobile across the sectors, so that the rental rate is equalized across the sectors. With $\theta \to \infty$, it reduces to the one-sector RBC model, which is widely used in the previous literature to study the welfare cost of business cycles. The model is almost similar to our benchmark model except the law of motion for capital stock:

[6] While not presented here, the welfare cost is still positive even when $\eta = 10$.

$$K_{t+1} = I_t \left(1 - \phi\left(\frac{I_t}{I_{t-1}}\right)\right) + (1-\delta)K_t, \tag{4.1}$$

where $K_t = K_{c,t} + K_{i,t}$ and $I_t = I_{c,t} + I_{i,t}$. In addition, the rental rates are the same across the sectors. i.e. $R_{c,t} = R_{i,t}$.

In the last economy, Economy 3, it is almost impossible to alter the rate of utilization of physical capital. Hence, the friction in using the physical capital is the greatest in this economy. Therefore, the real friction in move capital across sectors becomes substantial as we move from Economy 1 to Economy 3.

For the experiment, we change the value of θ and κ in each economy and then compute the welfare cost in each case. Table 4.4 to 4.6 show the results.

Table 4.4: Welfare Cost of Business Cycles in Economy 1: Economy with $\delta^2 = 0$

		κ				
		0.01	0.6	1	4	8
	1,000,000	−0.0047312	−0.0081937	−0.0047919	0.0023923	0.0044048
	1,000	−0.0046672	−0.0081653	−0.0047747	0.0023954	0.0044059
θ	0.3012	0.0050209	0.001827	0.0018632	0.0036513	0.0048227
	0.0001	0.0051599	0.0030654	0.0028719	0.0038436	0.0048383
	0.000001	0.0051599	0.003707	0.0028723	0.0038437	0.0048383

Note: In each experiment, we fix other parameter values to the benchmark case.

Table 4.5: Welfare Cost of Business Cycles in Economy 2: Baseline Economy

		κ				
		0.01	0.6	1	4	8
	1,000,000	−0.0038667	−0.003044	−0.0012334	0.0026817	0.0038475
	1,000	−0.0038154	−0.0030263	−0.0012224	0.0026838	0.0038483
θ	0.3012	0.0040842	0.0031827	0.0031699	0.0037827	0.0042588
	0.0001	0.004248	0.0039201	0.0038769	0.0041027	0.004377
	0.000001	0.0042481	0.0039203	0.0038772	0.0041029	0.0043771

Note: In each experiment, we fix other parameter values to the benchmark case. $\kappa_C = \kappa_I$ are assumed to be the same.

Several observations are noteworthy here. First, the welfare cost of business cycles becomes

Table 4.6: Welfare Cost of Business Cycles in Economy 3: Economy with $\delta^2 = 1,000,000$

		κ				
		0.01	0.6	1	4	8
	$1,000,000$	0.0016641	-0.00036141	0.00023727	0.0032372	0.00468
	$1,000$	0.0016771	-0.0003515	0.00024428	0.0032385	0.0046804
θ	0.3012	0.0050365	0.0042473	0.0039975	0.0039802	0.0046377
	0.0001	0.0052657	0.0050146	0.0048842	0.0043764	0.004232
	0.000001	0.0052658	0.0050149	0.0048846	0.0043767	0.0042318

Note: In each experiment, we fix other parameter values to the benchmark case. $\kappa_C = \kappa_I$ are assumed to be the same.

negative i.e. economic fluctuations are desirable as the sectoral factor immobility restrictions become less tight. Especially, keeping κ as constant, higher intersectoral labor substitutability (θ becomes higher) tends to decrease the welfare cost of business cycle in any economy. As θ becomes very high, the welfare cost can become negative. Moreover, as the friction to change the rate of capital utilization becomes lower (δ^2 becomes lower), the chance of obtaining negative welfare cost of business cycles increases. In summary, more volatile environment can be welfare-improving also in our two-sector model when the sectoral capital immobility becomes lower and the secotral labor substitutability becomes higher. However, the negative welfare cost of business cycles is possible in our model when parameter values are not plausible to reproduce labor market dynamics. For instance, the sectoral labor comovement is not plausible in the economy when the welfare cost of business cycles is negative. Even when the labor is almost perfectly substitutes with labor in other sector (when θ is extremely high), certain degree of κ, whether it is very low or relatively high, ensures that the welfare cost of business cycles is positive. On the other hand, as long as the degree of intersectoral labor substitutability is low enough, which is more favored by the data, removing business cycles is always beneficial.

Second, decreasing labor substitutability and increasing capital immobility tend to increase welfare cost of business cycles. This is consistent with the usual intuition; as the frictions in the economy, which prevents allocating resources from one sector to other sector, becomes higher, the volatile environment becomes more costly. Hence, it is important to consider the appropriate degree of frictions in the economy when evaluating the cost from the volatility.

Lastly, we provide our own explanation why the welfare cost of business cycles is somewhat lower in our model than that of Otrok (2001a). Notice that θ is infinite in Otrok (2001a)'s model. Hence, it yields lower welfare cost of business cycles. However, the capital is perfectly immobile in his model. Hence, our findings from Table 4.4 to 4.6 imply that the welfare cost is not necessarily lower in his model. In addition, lower σ and higher η in our model than what Otrok (2001a) used implies that the welfare cost is lower in our model, which is the observation from Table 4.3. On the other hand, h is lower (both parameters in our model and his model are in the region where the welfare cost is decreasing in h) in our model hence the welfare cost is higher in our model. In summary, there is no definite reason why our welfare cost metric is higher than what Otrok (2001a) used while the intersectoral immobility is greater in our model economy.

4.3 Other Shocks

In this section, we present the results when exogenous shock to the economy are (1) discount factor shocks (b_t) and (2) investment-specific technology shocks (Z_{it}).

Discount Factor Shock. Here, we assume that the exogenous shock in the model economy is the shock to the discount factor of the consumer. Parameters are the same with the benchmark case and $\rho_b = 0.94$ and $\sigma_b = 0.01$. The experiment we conduct here is changing the value of θ while keeping other parameters as the same.

Table 4.7: Welfare Cost of Business Cycles: Discount Factor Shock

θ	$1,000,000$	$1,000$	10	0.3012	0.01	0.0001
Welfare Cost (%)	0.00106	0.00107	0.00114	0.00118	0.00109	0.00108

Most importantly, the welfare costs are positive regardless of the choice of θ when the discount factor shock is the major source of the fluctuations. Hence, the more volatile environment is less preferred by consumers in the economy where demand-side disturbance distorts the incentive of the consumers to substitute intertemporally without supply-side shocks. This is because the discount factor shock itself does not affect the productivity of this economy so that mean effect from the fluctuations is not big.

Investment-Specific Technology Shock. We now assume that the exogenous shock in the model economy is the shock to investment. Again, parameters are the same with the benchmark case and $\rho_i = 0.947$ and $\sigma_{Z,i} = 0.01$. We conduct the same experiment by changing the value of θ:

Table 4.8: Welfare Cost of Business Cycles: Investment-Specific Technology Shock

θ	$1,000,000$	$1,000$	10	0.3012	0.01	0.0001
Welfare Cost (%)	-0.00314	-0.00314	-0.00272	-0.00105	-0.00059	-0.00056

First, the welfare costs are always negative when there only exists an investment-specific technology shock, which is consistent with the finding in Lester, Pries, and Sims (2014). They also find that the investment-specific technology shock enlarges the parameter region where the welfare cost of business cycle becomes negative. The intuition is that the shock directly affects the efficiency of investment; now the labor reallocation to investment sector for consumption smoothing is so desirable for consumers when compared to the economy where the aggregate TFP shock is the source of the fluctuations. As a result, the effect of the fluctuations through which more volatile economy can be preferred by consumers (Cho, Cooley, and Kim (2015) and Lester, Pries, and Sims (2014)) dominates the effect of the intersectoral labor substitutability that we find.

Second, while the welfare cost is negative, the finding that higher θ can lower the estimated welfare cost of business cycles is still observed in this economy.

5 Conclusion

In this paper, we calculate the welfare cost of business cycles with the two-sector business cycle model whose degree of intersectoral labor substitutability can be varied. The estimated welfare cost from our model suggests the possibility that welfare cost of business cycles can be underestimated in the (one-sector) model that has been widely used in the previous literature. In particular, the welfare cost of business cycles is positive under our favorable parameterization.

Hence, our finding indicates the importance of incorporating realistic features of the labor market when analyzing the welfare cost of business cycles.

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A Appendix A: Simple Model

The first order conditions are

$$\begin{aligned} Z_{1t}^{1-\gamma} N_{1t}^{-\gamma} &= N_t^{\frac{1}{\eta}-\frac{1}{\theta}} N_{1t}^{\frac{1}{\theta}} && \text{(A.1)} \\ Z_{2t}^{1-\gamma} N_{2t}^{-\gamma} &= N_t^{\frac{1}{\eta}-\frac{1}{\theta}} N_{2t}^{\frac{1}{\theta}} && \text{(A.2)} \end{aligned}$$

where $N_t^{-\frac{1}{\theta}} = \left[N_{1t}^{\frac{\theta+1}{\theta}} + N_{2t}^{\frac{\theta+1}{\theta}}\right]^{-\frac{1}{\theta+1}}$.

Dividing equation (A.1) by equation (A.2) yields

$$\left(\frac{N_{1t}}{N_{2t}}\right)^{\frac{1}{\theta}+\gamma} = \left(\frac{Z_{1t}}{Z_{2t}}\right)^{1-\gamma} \tag{A.3}$$

Hence,

$$N_{1t} = \left(\frac{Z_{1t}}{Z_{2t}}\right)^{\frac{1-\gamma}{\gamma+\frac{1}{\theta}}} N_{2t} \tag{A.4}$$

Then $N_t = \left[N_{1t}^{\frac{\theta+1}{\theta}} + N_{2t}^{\frac{\theta+1}{\theta}}\right]^{\frac{\theta}{\theta+1}}$ implies $N_t = \kappa\left(\frac{Z_{1t}}{Z_{2t}}\right) N_{2t}$ where $\kappa\left(\frac{Z_{1t}}{Z_{2t}}\right) \equiv \left[\left(\frac{Z_{1t}}{Z_{2t}}\right)^{\frac{(1+\theta)(1-\gamma)}{\gamma\theta+1}} + 1\right]^{\frac{\theta}{\theta+1}}$. Notice that $\kappa(\cdot)$ is increasing in Z_{1t} when $0 < \gamma < 1$ but is decreasing in Z_{1t} when $\gamma > 1$.

Substituting this relationship into the first order condition (A.2). Then we can obtain the solution for labor inputs as in equation (2.3) – (2.5).

A.1 Sectoral Labor Comovement in Simple Economy

In this section, we study how the changes in θ affect the prediction of the model on sectoral labor comovement problem (consistency with the data). Here we only state the result when $\gamma \in (0,1)$, but the same condition holds for the sectoral labor comovement when $\gamma > 1$.

Proposition 3 (Sectoral Labor Comovement in the Simple Model)**.** *Suppose that $\gamma \in (0,1)$. Assume that there is a positive technology shock in sector 1. i.e. Z_{1t} increases while Z_{2t} remains same. Then the followings hold.*

- *Suppose that $\theta > \eta$ holds. Hence, the degree of intersectoral labor substitutability is high. Then N_{1t} and N_t increase while N_{2t} decreases for given positive shock to Z_{1t}.*
- *Suppose that $\theta < \eta$ holds. Hence, the degree of intersectoral labor substitutability is low. Then N_{1t}, N_{2t}, and N_t all increase for given positive shock to Z_{1t}.*

Proof. From the relationship between $\kappa(\cdot)$ and Z_t, equation (2.4), and equation (2.5). □

Therefore, the simple model can explain the sectoral labor comovement problem as a function θ; if intersectoral labor substitutability is relatively high (θ is high), the sectoral comovement problem arises. On the other hand, if it is costly to move hours from one sector to other sector (θ is low), the sectoral comovement problem disappears. Suppose that labor is relatively freely mobile across sectors. Then given positive shock in sector 1, the consumer reallocates a fraction of hours from producing c_{2t} into producing c_{1t} in order to exploit high productivity in sector 1. As a result, the sectoral labor comovement problem arises. On the other hand, if it incurs some costs when reallocating hours across the sectors, she cannot decrease N_{2t} as she wants, which resolve the sectoral labor comovement problem.

B Appendix B: Equilibrium conditions

In this appendix, we provide the equilibrium conditions of the stationary economy.

B.1 Stationarizing the model

Since A_t, V_t have their trend respectively, we need to stationarize the model. The following shows the growth rate on balanced growth path.

- The growth rate of A_t=z
- The growth rate of V_t=v
- The growth rate of N_t, $N_{c,t}$, $N_{i,t}$, $u_{c,t}$, $u_{i,t}$ $= 1$
- The growth rate of $R_{c,t}/P_{c,t}$, $R_{i,t}/P_{c,t}$, $P_{i,t}/P_{c,t}$ $= v^{-1}$
- The growth rate of C_t, $W_{c,t}/P_{c,t}$, $W_{i,t}/P_{c,t}$ $= z^{\frac{1}{1-\alpha}} v^{\frac{\alpha}{1-\alpha}}$
- The growth rate of I_t, $I_{c,t}$, $I_{i,t}$, $K_{c,t}$, $K_{i,t}$ $= z^{\frac{1}{1-\alpha}} v^{\frac{1}{1-\alpha}}$
- The growth rate of Λ_t $= z^{\frac{-1}{(1-\alpha)\sigma}} v^{\frac{-\alpha}{(1-\alpha)\sigma}}$
- The growth rate of $\Gamma_{c,t}$, $\Gamma_{i,t}$ $= z^{\frac{-1}{(1-\alpha)\sigma}} v^{\frac{-\alpha}{(1-\alpha)\sigma}-1}$

So, we need to transform the following variables:

$$k_{c,t+1} = \frac{K_{c,t+1}}{A_t^{\frac{1}{1-\alpha}} V_t^{\frac{1}{1-\alpha}}}, \quad k_{i,t+1} = \frac{K_{i,t+1}}{A_t^{\frac{1}{1-\alpha}} V_t^{\frac{1}{1-\alpha}}}, \quad c_t = \frac{C_t}{A_t^{\frac{1}{1-\alpha}} V_t^{\frac{\alpha}{1-\alpha}}},$$

$$i_t = \frac{I_t}{A_t^{\frac{1}{1-\alpha}} V_t^{\frac{1}{1-\alpha}}}, \quad i_{c,t} = \frac{I_{c,t}}{A_t^{\frac{1}{1-\alpha}} V_t^{\frac{1}{1-\alpha}}}, \quad i_{i,t} = \frac{I_{i,t}}{A_t^{\frac{1}{1-\alpha}} V_t^{\frac{1}{1-\alpha}}}$$

$$w_{c,t} = \frac{W_{c,t}}{P_{c,t}} \left(\frac{1}{A_t^{\frac{1}{1-\alpha}} V_t^{\frac{\alpha}{1-\alpha}}} \right), \quad w_{i,t} = \frac{W_{i,t}}{P_{c,t}} \left(\frac{1}{A_t^{\frac{1}{1-\alpha}} V_t^{\frac{\alpha}{1-\alpha}}} \right)$$

$$\lambda_t = \Lambda_t A_t^{\frac{1}{(1-\alpha)\sigma}} V_t^{\frac{\alpha}{(1-\alpha)\sigma}}, \quad \tau_{c,t} = \Gamma_{c,t} A_t^{\frac{1}{(1-\alpha)\sigma}} V_t^{\frac{\alpha}{(1-\alpha)\sigma}+1}, \quad \tau_{i,t} = \Gamma_{i,t} A_t^{\frac{1}{(1-\alpha)\sigma}} V_t^{\frac{\alpha}{(1-\alpha)\sigma}+1}$$

$$p_t = \frac{P_{i,t}}{P_{c,t}} V_t, \quad r_{c,t} = \frac{R_{c,t}}{P_{c,t}} V_t, \quad r_{i,t} = \frac{R_{i,t}}{P_{c,t}} V_t$$

For simplicity, we use following auxiliary variables

$$L_t^C = A_t^{\frac{1}{1-\alpha}} V_t^{\frac{\alpha}{1-\alpha}}$$

$$G_t^A = \frac{A_t}{A_{t-1}}, \quad G_t^V = \frac{V_t}{V_{t-1}}, \quad G_t^C = \frac{L_t^C}{L_{t-1}^C} = \left(\frac{A_t}{A_{t-1}}\right)^{\frac{1}{1-\alpha}} \left(\frac{V_t}{V_{t-1}}\right)^{\frac{\alpha}{1-\alpha}} = \left(G_t^A\right)^{\frac{1}{1-\alpha}} \left(G_t^V\right)^{\frac{\alpha}{1-\alpha}}$$

$$k_{c,t+1} = \frac{K_{c,t+1}}{L_t^C V_t}, \quad k_{i,t+1} = \frac{K_{i,t+1}}{L_t^C V_t}, \quad c_t = \frac{C_t}{L_t^C}, \quad i_t = \frac{I_t}{L_t^C V_t}, \quad i_{c,t} = \frac{I_{c,t}}{L_t^C V_t}, \quad i_{i,t} = \frac{I_{i,t}}{L_t^C V_t}$$

$$w_{c,t} = \frac{W_{c,t}}{P_{c,t}} \left(\frac{1}{L_t^C}\right), \quad w_{i,t} = \frac{W_{i,t}}{P_{c,t}} \left(\frac{1}{L_t^C}\right)$$

$$\lambda_t = \Lambda_t \left(L_t^C\right)^{\frac{1}{\sigma}}, \quad \tau_{c,t} = \Gamma_{c,t} \left(L_t^C\right)^{\frac{1}{\sigma}} V_t, \quad \tau_{i,t} = \Gamma_{i,t} \left(L_t^C\right)^{\frac{1}{\sigma}} V_t$$

$$p_t = \frac{P_{i,t}}{P_{c,t}} V_t, \quad r_{c,t} = \frac{R_{c,t}}{P_{c,t}} V_t, \quad r_{i,t} = \frac{R_{i,t}}{P_{c,t}} V_t$$

B.1.1 Transformed system

- **Consumption demand**

$$b_t \left(c_t - h\frac{c_{t-1}}{G_t^C}\right)^{-\frac{1}{\sigma}} \left(1 + \left(\frac{1}{\sigma} - 1\right) v(N_t)\right)^{\frac{1}{\sigma}} - (G_{t+1}^C)^{-\frac{1}{\sigma}} \beta h b_{t+1} \left(c_{t+1} - h\frac{c_t}{G_{t+1}^C}\right)^{-\frac{1}{\sigma}} \left(1 + \left(\frac{1}{\sigma} - 1\right) v(N_{t+1})\right)^{\frac{1}{\sigma}} = \lambda_t \tag{B.1}$$

- **Labor supply to consumption sector**

$$b_t \frac{1}{\sigma} \left(c_t - h\frac{c_{t-1}}{G_t^C}\right)^{1-\frac{1}{\sigma}} \left(1 + \left(\frac{1}{\sigma} - 1\right) v(N_t)\right)^{\frac{1}{\sigma}-1} v^{'}(N_t) \frac{\partial N_t}{\partial N_{c,t}} = \lambda_t w_{c,t} \tag{B.2}$$

where $\frac{\partial N_t}{\partial N_{c,t}} = N_t^{-\frac{1}{\theta}} N_{c,t}^{\frac{1}{\theta}}$.

- **Labor supply to investment sector**

$$b_t \frac{1}{\sigma} \left(c_t - h\frac{c_{t-1}}{G_t^C}\right)^{1-\frac{1}{\sigma}} \left(1 + \left(\frac{1}{\sigma} - 1\right) v(N_t)\right)^{\frac{1}{\sigma}-1} v^{'}(N_t) \frac{\partial N_t}{\partial N_{i,t}} = \lambda_t w_{i,t} \tag{B.3}$$

where $\frac{\partial N_t}{\partial N_{i,t}} = N_t^{-\frac{1}{\theta}} N_{i,t}^{\frac{1}{\theta}}$.

- **Capital service supply to consumption sector**

$$\lambda_t r_{c,t} = \tau_{c,t}\delta^{'}(u_{c,t}) = \tau_{c,t}(\delta_c^1 + \delta_c^2(u_{c,t}-1)) \tag{B.4}$$

- **Capital service supply to investment sector**

$$\lambda_t r_{i,t} = \tau_{i,t}\delta^{'}(u_{i,t}) = \tau_{i,t}(\delta_i^1 + \delta_i^2(u_{i,t}-1)) \tag{B.5}$$

- **Capital Euler equation in consumption sector**

$$\tau_{c,t}(G_{t+1}^C)^{\frac{1}{\sigma}} G_{t+1}^V = \beta E_t\left[\lambda_{t+1} r_{c,t+1} u_{c,t+1} + \tau_{c,t+1}(1-\delta(u_{c,t+1}))\right] \tag{B.6}$$

- **Capital Euler equation in investment sector**

$$\tau_{i,t}(G_{t+1}^C)^{\frac{1}{\sigma}} G_{t+1}^V = \beta E_t\left[\lambda_{t+1} r_{i,t+1} u_{i,t+1} + \tau_{i,t+1}(1-\delta(u_{i,t+1}))\right] \tag{B.7}$$

- **Investment in consumption sector**

$$\begin{aligned}\lambda_t p_t = {} & \tau_{c,t}\left[1 - \frac{\kappa_c}{2}\left(\frac{i_{c,t}}{i_{c,t-1}} G_t^C G_t^V - G^C G^V\right)^2 - \kappa_c\left(\frac{i_{c,t}}{i_{c,t-1}} G_t^C G_t^V - G^C G^V\right)\frac{i_{c,t}}{i_{c,t-1}} G_t^C G_t^V\right] \\ & + \beta E_t\left[(G_{t+1}^C)^{2-\frac{1}{\sigma}} G_{t+1}^V \tau_{c,t+1}\kappa_c\left(\frac{i_{c,t+1}}{i_{c,t}} G_{t+1}^C G_{t+1}^V - G^C G^V\right)\left(\frac{i_{c,t+1}}{i_{c,t}}\right)^2\right]\end{aligned} \tag{B.8}$$

- **Investment in investment sector**

$$\begin{aligned}\lambda_t p_t = {} & \tau_{i,t}\left[1 - \frac{\kappa_i}{2}\left(\frac{i_{i,t}}{i_{i,t-1}} G_t^C G_t^V - G^C G^V\right)^2 - \kappa_c\left(\frac{i_{i,t}}{i_{i,t-1}} G_t^C G_t^V - G^C G^V\right)\frac{i_{i,t}}{i_{i,t-1}} G_t^C G_t^V\right] \\ & + \beta E_t\left[(G_{t+1}^C)^{2-\frac{1}{\sigma}} G_{t+1}^V \tau_{i,t+1}\kappa_i\left(\frac{i_{i,t+1}}{i_{i,t}} G_{t+1}^C G_{t+1}^V - G^C G^V\right)\left(\frac{i_{i,t+1}}{i_{i,t}}\right)^2\right]\end{aligned} \tag{B.9}$$

- **Labor demand in consumption sector**

$$w_{c,t} = (1-\alpha)\frac{c_t}{N_{c,t}} \tag{B.10}$$

- **Labor demand in investment sector**

$$\frac{w_{i,t}}{p_t} = (1-\alpha)\frac{i_t}{N_{i,t}} \tag{B.11}$$

- **Capital services demand in consumption sector**

$$r_{c,t} = \alpha\frac{c_t}{u_{c,t}k_{c,t}} G_t^C G_t^V \tag{B.12}$$

- **Capital services demand in investment sector**

$$\frac{r_{i,t}}{p_t} = \alpha \frac{i_t}{u_{i,t}k_{i,t}} G_t^C G_t^V \tag{B.13}$$

- **Production function in consumption sector**

$$c_t = (G_t^A G_t^V)^{-\frac{\alpha}{1-\alpha}} a_t z_{c,t} (u_{c,t}k_{c,t})^{\alpha} (N_{c,t})^{1-\alpha} \tag{B.14}$$

- **Production function in investment sector**

$$i_t = (G_t^A G_t^V)^{-\frac{\alpha}{1-\alpha}} a_t z_{i,t} (u_{i,t}k_{i,t})^{\alpha} (N_{i,t})^{1-\alpha} \tag{B.15}$$

- **Aggregate labor index**

$$N_t = \left[N_{c,t}^{\frac{\theta+1}{\theta}} + N_{i,t}^{\frac{\theta+1}{\theta}} \right]^{\frac{\theta}{\theta+1}} \tag{B.16}$$

- **Aggregate investment**

$$i_t = i_{c,t} + i_{i,t} \tag{B.17}$$

- **Capital accumulation in consumption sector**

$$k_{c,t+1} = i_{c,t} \left[1 - \frac{\kappa_c}{2} \left(\frac{i_{c,t}}{i_{c,t-1}} G_t^C G_t^V - G^C G^V \right)^2 \right] + [1 - \delta(u_{c,t})] k_{c,t} (G_t^C G_t^V)^{-1} \tag{B.18}$$

- **Capital accumulation in investment sector**

$$k_{i,t+1} = i_{i,t} \left[1 - \frac{\kappa_i}{2} \left(\frac{i_{i,t}}{i_{i,t-1}} G_t^C G_t^V - G^C G^V \right)^2 \right] + [1 - \delta(u_{i,t})] k_{i,t} (G_t^C G_t^V)^{-1} \tag{B.19}$$

C Appendix C: Derivation of Equation (3.23)

Since the equilibrium is second-order approximated following Schmitt-Grohé and Uribe (2004), we restrict our attention to the second-order approximated form of λ as well (see Schmitt-Grohé and Uribe (2006)). In equilibrium, both of V^{NF} and V^F are functions of the initial steady-state vector x_0 and the parameter σ_z. Define

$$\lambda(x_0, \sigma_z) = \Lambda(x_0, \sigma_z) \equiv \left(\frac{V^{NF}(x_0, \sigma_z)}{V^F(x_0, \sigma_z)} \right)^{\frac{\sigma}{\sigma-1}} - 1 \tag{C.1}$$

As discussed in Schmitt-Grohé and Uribe (2006), it is sufficient to approximate the above expression upto the second-order only with respect to the scaling parameter, σ_z, around the initial steady state $x_0 = x$ and $\sigma_z = 0$.

$$\lambda(x, 0) \approx \frac{\Lambda_{\sigma_z \sigma_z}(x, 0)}{2} \sigma_z^2 \tag{C.2}$$

since $\Lambda(x, 0) = 0$ and $\Lambda_{\sigma_z}(x, 0) = 0$.

We now derive the expression $\Lambda_{\sigma_z \sigma_z}(x_0, \sigma_z)$. From now on, for convenience, (x_0, σ_z) will be dropped until it is necessary. We first differentiate the right hand side of the equation (3.22) with respect to σ_z:

$$\Lambda_{\sigma_z} = \frac{\sigma}{\sigma-1}\left(\frac{V^{NF}}{V^F}\right)^{\frac{\sigma}{\sigma-1}-1}\frac{V^{NF}_{\sigma_z}V^F - V^{NF}V^F_{\sigma_z}}{(V^F)^2} \tag{C.3}$$

Then

$$\begin{aligned}\Lambda_{\sigma_z\sigma_z} &= \frac{\sigma}{\sigma-1}\left(\frac{\sigma}{\sigma-1}-1\right)\left(\frac{V^{NF}}{V^F}\right)^{\frac{\sigma}{\sigma-1}-2}\left(\frac{V^{NF}_{\sigma_z}V^F - V^{NF}V^F_{\sigma_z}}{(V^F)^2}\right)^2 \\ &+ \frac{\sigma}{\sigma-1}\left(\frac{V^{NF}}{V^F}\right)^{\frac{\sigma}{\sigma-1}-1}\frac{(V^{NF}_{\sigma_z\sigma_z}V^F + V^{NF}_{\sigma_z}V^F_{\sigma_z} - V^{NF}_{\sigma_z}V^F_{\sigma_z} - V^{NF}V^F_{\sigma_z\sigma_z})(V^F)^2 - 2(V^{NF}_{\sigma_z}V^F - V^{NF}V^F_{\sigma_z})V^F V^F_{\sigma_z}}{(V^F)^4}\end{aligned} \tag{C.4}$$

At $(x_0, \sigma_z) = (x, 0)$, the above expression reduces to the following expression.

$$\Lambda_{\sigma_z\sigma_z} = \frac{\sigma}{\sigma-1}\frac{V^{NF}_{\sigma_z\sigma_z} - V^F_{\sigma_z\sigma_z}}{V^F} \tag{C.5}$$

Hence,

$$\lambda(x,0) \approx \frac{\sigma}{\sigma-1}\frac{V^{NF}_{\sigma_z\sigma_z}(x,0) - V^F_{\sigma_z\sigma_z}(x,0)}{V^F(x,0)}\frac{\sigma_z^2}{2} \tag{C.6}$$

which is in principle equivalent to the equation (39) in Schmitt-Grohé and Uribe (2006).

Notice that non-fluctuating economy in our model is at the steady-state hence $V^{NF}_{\sigma_z\sigma_z}(x,0) = 0$. Therefore, it suffices to check the following equation:

$$\lambda(x,0) \approx \frac{\sigma}{1-\sigma}\frac{V^F_{\sigma_z\sigma_z}(x,0)}{V^F(x,0)}\frac{\sigma_z^2}{2} \tag{C.7}$$

If the non-stochastic economy is at the steady-state, the following holds (see Schmitt-Grohé and Uribe (2004)):

$$\ln C_t = \ln c + \frac{1}{2}c_{\sigma_z\sigma_z}\sigma_z^2 \quad\text{and}\quad \ln N_t = \ln n + \frac{1}{2}n_{\sigma_z\sigma_z}\sigma_z^2 \tag{C.8}$$

where c and n are steady-state variables. The above relationship holds for every t hence we denote $C_t = C$ and $N_t = N$ in what follows. Then the equation (3.20) would be

$$V^F = \underbrace{\frac{(1-h)^{1-\frac{1}{\sigma}}}{(1-\beta)\left(1-\frac{1}{\sigma}\right)}}_{\equiv\Omega} C^{1-\frac{1}{\sigma}}\left(1+(\frac{1}{\sigma}-1)V(N)\right)^{\frac{1}{\sigma}} \tag{C.9}$$

Then

$$V^F_{\sigma_z} = \Omega\left[\left(1-\frac{1}{\sigma}\right)C^{-\frac{1}{\sigma}}\frac{dC}{d\sigma_z}\left(1+\left(\frac{1}{\sigma}-1\right)V(N)\right)^{\frac{1}{\sigma}} + \frac{1}{\sigma}\left(\frac{1}{\sigma}-1\right)C^{1-\frac{1}{\sigma}}\left(1+\left(\frac{1}{\sigma}-1\right)V(N)\right)^{\frac{1}{\sigma}-1}V'(N)\frac{dN}{d\sigma_z}\right] \tag{C.10}$$

and

$$\begin{aligned}V^F_{\sigma_z\sigma_z}/\Omega =& \left(1-\frac{1}{\sigma}\right)\left(-\frac{1}{\sigma}\right)C^{-\frac{1}{\sigma}-1}\left(\frac{dC}{d\sigma_z}\right)^2\left(1+\left(\frac{1}{\sigma}-1\right)V(N)\right)^{\frac{1}{\sigma}}\\ &+\left(1-\frac{1}{\sigma}\right)C^{-\frac{1}{\sigma}}\frac{d^2C}{d\sigma_z^2}\left(1+\left(\frac{1}{\sigma}-1\right)V(N)\right)^{\frac{1}{\sigma}}\\ &+2\frac{1}{\sigma}\left(1-\frac{1}{\sigma}\right)\left(\frac{1}{\sigma}-1\right)C^{-\frac{1}{\sigma}}\frac{dC}{d\sigma_z}\left(1+\left(\frac{1}{\sigma}-1\right)V(N)\right)^{\frac{1}{\sigma}-1}V'(N)\frac{dN}{d\sigma_z}\\ &+\frac{1}{\sigma}\left(\frac{1}{\sigma}-1\right)^3C^{1-\frac{1}{\sigma}}\left(1+\left(\frac{1}{\sigma}-1\right)V(N)\right)^{\frac{1}{\sigma}-2}(V'(N))^2\left(\frac{dN}{d\sigma_z}\right)^2\\ &+\frac{1}{\sigma}\left(\frac{1}{\sigma}-1\right)C^{1-\frac{1}{\sigma}}\left(1+\left(\frac{1}{\sigma}-1\right)V(N)\right)^{\frac{1}{\sigma}-1}V''(N)\left(\frac{dN}{d\sigma_z}\right)^2\\ &+\frac{1}{\sigma}\left(\frac{1}{\sigma}-1\right)C^{1-\frac{1}{\sigma}}\left(1+\left(\frac{1}{\sigma}-1\right)V(N)\right)^{\frac{1}{\sigma}-1}V'(N)\frac{d^2N}{d\sigma_z^2}\end{aligned} \tag{C.11}$$

Notice that at the steady-state with $\sigma_z = 0$, $dC/d\sigma_z = dN/d\sigma_z = 0$. As a result,

$$V^F_{\sigma_z\sigma_z} = \Omega\left[\left(1-\frac{1}{\sigma}\right)C^{-\frac{1}{\sigma}}\frac{d^2C}{d\sigma_z^2}\left(1+\left(\frac{1}{\sigma}-1\right)V(N)\right)^{\frac{1}{\sigma}}+\frac{1}{\sigma}\left(\frac{1}{\sigma}-1\right)C^{1-\frac{1}{\sigma}}\left(1+\left(\frac{1}{\sigma}-1\right)V(N)\right)^{\frac{1}{\sigma}-1}V'(N)\frac{d^2N}{d\sigma_z^2}\right] \tag{C.12}$$

One can show $\frac{d^2C}{d\sigma_z^2} = c_{\sigma_z\sigma_z}C$ and $\frac{d^2N}{d\sigma_z^2} = n_{\sigma_z\sigma_z}N$ at $\sigma_z = 0$. Hence,

$$V^F_{\sigma_z\sigma_z} = \frac{(1-h)^{1-\frac{1}{\sigma}}}{(1-\beta)}C^{1-\frac{1}{\sigma}}\left(1+\left(\frac{1}{\sigma}-1\right)V(N)\right)^{\frac{1}{\sigma}}\left[c_{\sigma_z\sigma_z}-\frac{V'(N)N}{\sigma\left(1+\left(\frac{1}{\sigma}-1\right)V(N)\right)}n_{\sigma_z\sigma_z}\right] \tag{C.13}$$

Hence,

$$\frac{V^F_{\sigma_z\sigma_z}(x,0)}{V^F(x,0)} = \left(1-\frac{1}{\sigma}\right)\left[c_{\sigma_z\sigma_z}-\frac{V'(N)N}{\sigma\left(1+\left(\frac{1}{\sigma}-1\right)V(N)\right)}n_{\sigma_z\sigma_z}\right] \tag{C.14}$$

Finally, the equation (C.7) is now

$$\begin{aligned}\lambda(x,0) &\approx \frac{\sigma}{1-\sigma}\frac{V^F_{\sigma_z\sigma_z}(x,0)}{V^F(x,0)}\frac{\sigma_z^2}{2}\\ &= \left[\frac{V'(N)N}{\sigma\left(1+\left(\frac{1}{\sigma}-1\right)V(N)\right)}n_{\sigma_z\sigma_z}-c_{\sigma_z\sigma_z}\right]\frac{\sigma_z^2}{2}\end{aligned} \tag{C.15}$$

D Appendix D: Additional Figure

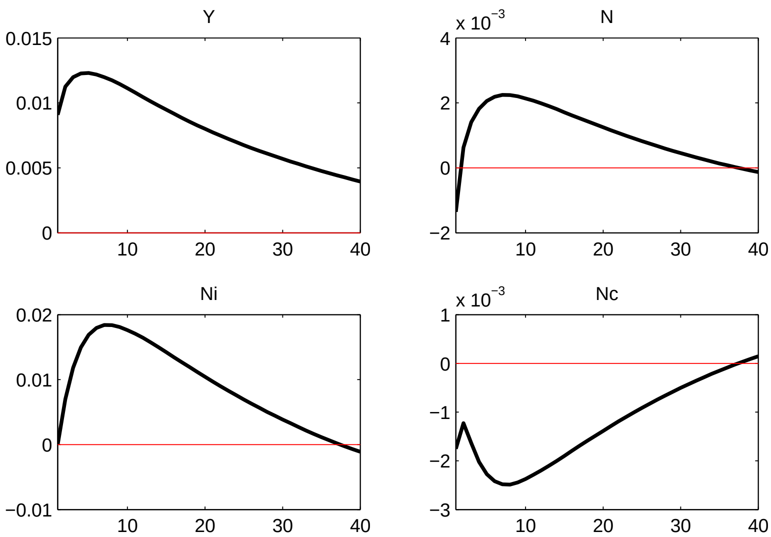

Figure D.1: Intersectoral Labor Comovement: Model with Perfect Labor Substitutability ($\theta = 1,000,000$)

Note: Horizontal axes take model periods and vertical axes measure percentage deviations from the steady-state values. Y, N, Ni, and Nc denote output, aggregate labor hours, labor hours devoted in investment sector, and labor hours devoted in consumption sector, respectively.

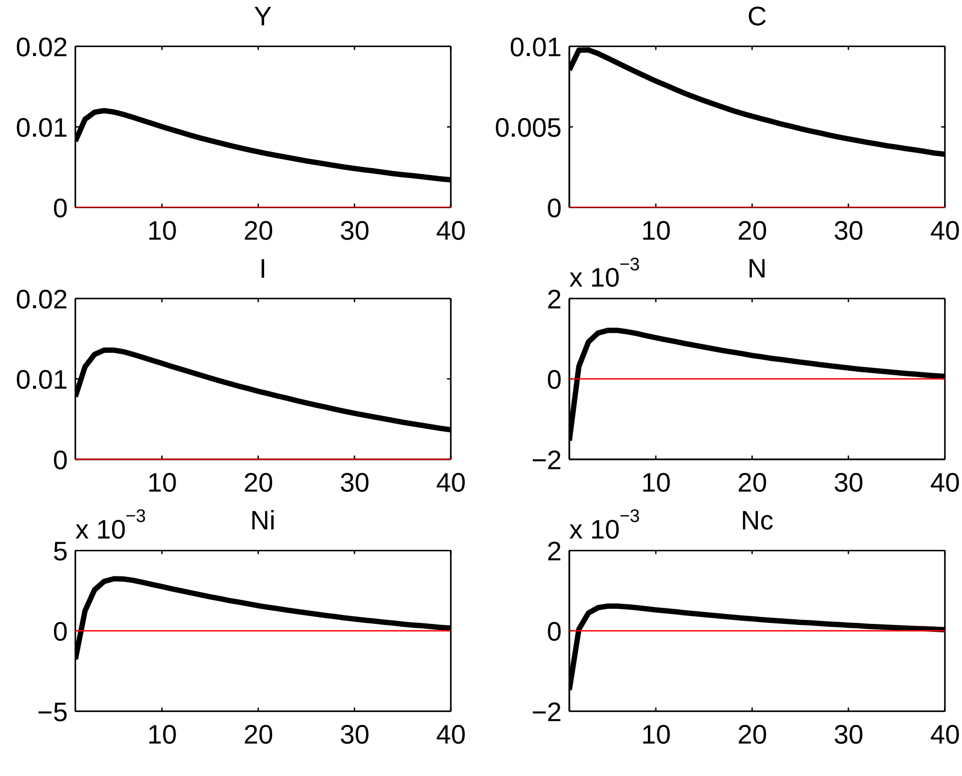

Figure D.2: Intersectoral Labor Comovement: Model with Imperfect Labor Substitutability ($\theta = 0.3014$)

Note: Horizontal axes take model periods and vertical axes measure percentage deviations from the steady-state values. Y, C, I, N, Ni, and Nc denote output, consumption, investment, aggregate labor hours, labor hours devoted in investment sector, and labor hours devoted in consumption sector, respectively.