Dynamics of Firms and Trade in General Equilibrium*

(very preliminary)

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Abstract

This paper develops a dynamic general equilibrium model that tries to reconcile the observation that aggregate movements of exports and imports are "disconnected" from real exchange rate movements, while firm-level exports co-move significantly with the real exchange rate. Firms are heterogenous, facing recurrent aggregate and firm-product specific productivity shocks, choose which goods to export, and decide to enter and exit the business endogenously. We calibrate and estimate the model with both aggregate and firm level data from Japan.

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1 Introduction

Figure 1 displays the series of aggregate real values of exports and imports together with the real exchange rate in Japan during the period of 1980-2009 in logarithmic scale. The real exchange rate is defined as the relative price between Japan's trading partners and Japan.[1] As the trading partners' goods become relatively more expensive, we expect that Japanese exports would increase and imports would decrease through substitution effect. However, such a relationship between trade and the real exchange rate is not evident in Figure 1. As Japanese real exchange rate depreciates, exports do not necessarily increase, and imports increase, which is not what we expect. During the entire sample period, the elasticity of exports with respect to the real exchange rate is -0.17, and that of imports is 0.08, although these estimates of elasticities are statistically insignificant. This lack of correlation, or correlation contrary to what we expect is an example of the so called "exchange rate disconnect puzzle," a long standing puzzle in international macroeconomics. This weak or opposite correlation between aggregate exports (or imports) and the exchange rate is observed in many other countries as well (see Hooper, Johnson, and Marquez (2000), and Dekle, Jeong, and Ryoo (2007)).[2] Obstfeld and Rogoff (2000) mention that the exchange rate disconnect puzzle is one of the major puzzles in the international macroeconomics.[3]

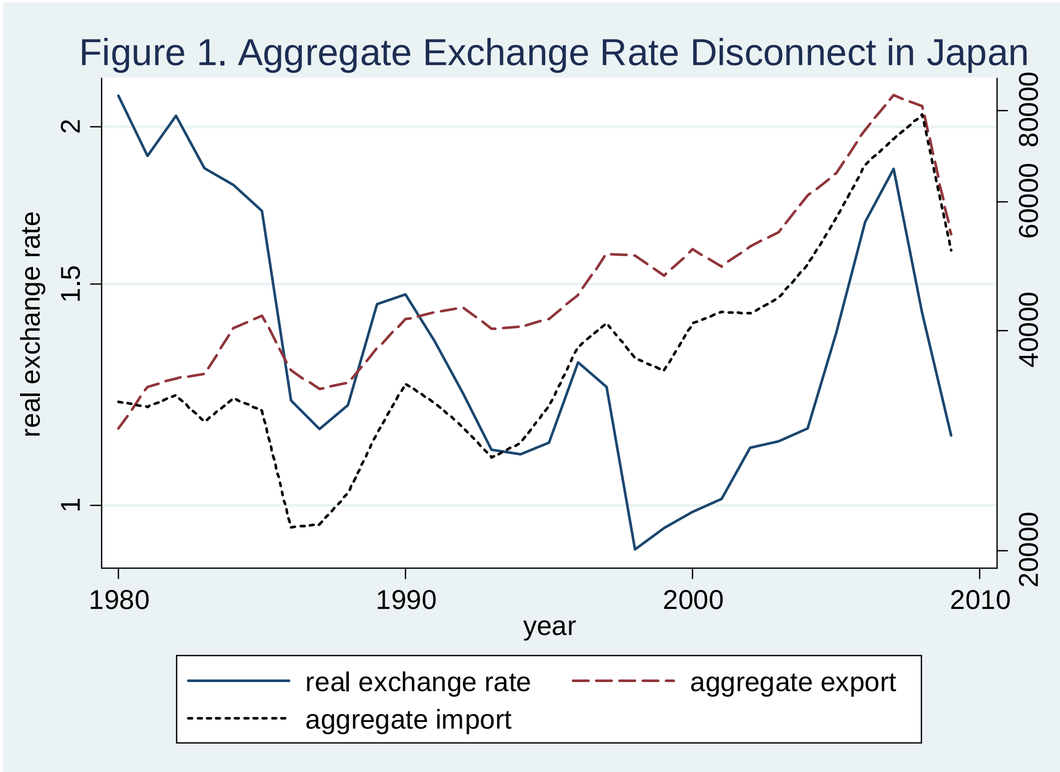
Interestingly, after the year 2000, Figure 1 shows that aggregate exports moved in the same direction with the real exchange rate, but aggregate imports also moved in the same direction. These co-movement during this period suggests that a general equilibrium linkage

[1] The real exchange rate is measured as the ratio of the weighted average of the prices of Japan's major trading partners (in yen term) to Japanese prices, where the weights are the trading shares. The four major trading partner countries included here are the U.S., European Union, South Korea, and China and their trading shares are 0.49, 0.366, 0.095, and 0.044, respectively. (Sources: OECD Statistics)

Aggregate real value of exports and imports are measured in billions of year 2000 yen using GDP deflator. (Source: Ministry of Treasury Trade Statistics: http://www.customs.go.jp/toukei/suii/html/time_e.htm)

[2] The list of other countries showing such weak correlation is Canada, France, Germany, Italy, the U.K., and the U.S. This empirical puzzle was first documented by Orcutt (1950).

[3] Note that this "exchange rate disconnect puzzle" is different from the so called "J-curve effect." The exchange rate disconnect puzzle is about the lack of association between the movements of exchange rates and *gross export quantities* while the J-curve effect is about the sluggish and J-shaped adjustment of trade balances (i.e., *net export sales*) in response to an improvement in the terms of trade. See Backus, Kehoe, and Kydland (1994) for the discussion of the J-curve effect.



may be important in order to understand the dynamics of trade and exchange rates in Japan, where intermediate goods trade is dominant in imports, and increasingly more important in exports.

Recent empirical studies using firm-level data have found a more robust relationship between export and the exchange rate. In contrast to the results using aggregate data, estimates using firm level tend to find a positive relationship between depreciating exchange rates and export quantities. Among other studies, Verhoogen (2008) finds that following the 1994 peso devaluation, Mexican firms increased their exports. Fitzgerald and Haller (2008), Dekle and Ryoo (2007), and Tybout and Roberts (1997) find a positive association between exports and exchange rate depreciation for Irish, Japanese and Colombian firms, respectively.

Some papers have tried to reconcile these aggregate and firm level results, but mostly in a partial equilibrium framework. Dekle, Jeong, and Ryoo (2007) show that in the aggregate export equation derived by consistently aggregating the firm level export equations, where industry level productivity and export share are controlled for, the disconnect puzzle disappears.

Berman, Martin, and Mayer (2009) use a model with heterogeneous firms in the spirit of Melitz (2003) to show that high productivity firms (who are heavily involved in exports) will raise their prices–that is, increase their markups–instead of increasing their export quantities in response to an exchange rate depreciation. The authors show that this selection effect of small quantity response of high productivity firms can explain the weak impact of exchange rate movements in aggregate data. There are some other recent papers that have tried to reconcile the discrepancy in a general equilibrium. Imbs and Majean (2009) and Feenstra, Russ, and Obstfeld (2010) show that the aggregation of heterogeneous industrial sectors can result in an aggregation bias in the elasticity of exports and imports with respect to exchange rates changes. Both of these papers examine only the steady-state.

In this paper, we develop a dynamic general equilibrium model with heterogeneous firms that attempts to reconcile the different responses of exports and imports to exchange rates at the aggregate- and at the firm-levels. Our model is a real business cycle model of a small open economy with a rich production structure. Firms are heterogeneous, facing recurrent aggregate and firm-product specific productivity shocks: they choose which varieties of goods to produce and export and decide to enter and exit endogenously. We calibrate and estimate our model with both aggregate and firm level data. We then carry out quantitative exercises regarding the impact of shocks to productivity and preferences on aggregate and firm-level exports and other variables of interest.[4]

We make a few choices to model heterogeneous firms to reflect our panel data of Japanese firms listed on the stock exchanges of Japan.[5] In a well-known paper, Melitz (2003) showed that,

[4]One distinguishing feature of our work is the inclusion of heterogeneous firm dynamics that is actually estimated from firm level data. In the estimation of the firm-level responses, in addition to the firm level data, we rely on the aggregate variables and moments generated from the general equilibrium model. Thus, in a sense, we provide a general equilibrium model that is integrated with a structural model of heterogeneous firm dynamics that is estimated from actual firm level data.

[5]The raw data used here and in our paper are from almost all of the firms listed on the stock exchanges of Japan.The particular data set that we use were compiled by the Development Bank of Japan (or "Kaigin," in Japanese prior to the 2008 re-organization of government-owned enterprises, when the name of the bank was changed). Japanese listed firms cover a fairly respectable portion of the entire Japanese economy in terms of output (Fukao, et. al., 2008). In 2000, the gross sales of all the firms listed on the stock exchanges of Japan were 81 percent of Japanese nominal GDP, and 60 percent of total sales in the Japanese economy. However, listed firms are larger than the average firm in the economy. Thus, the number of listed firms account for less than 12 percent of the total number of Japanese firms, and the number of employees in listed firms are only 40

when firms are heterogeneous in its total factor productivity and need to cover a fixed cost for export, only high productive and large firms export. Das, Roberts, and Tybout (2007) provide an empirical study showing that the difference in total factor productivity among producers explains whether they export or not, the so-called extensive margin of trade. In our Japanese panel data, there is a strong relationship between firm size and exporting status, as in Melitz (2003). The average total sales of the incumbent exporting firms is about twice as much as the non-exporting firms. When firms are different only because their total factor productivity are different, however, the share of export in total sales (export share) should be strongly correlated with firm size among the exporting firms (in addition to whether or not the firm exports at all). Our Japanese firm level data show that this prediction is not true. The correlation between the export share and total sales is rather weak. The average correlation coefficient is only 0.08 among all firms. Among exporting firms, the correlation coefficient becomes even lower at 0.05. This weak correlation remains robust even after controlling for the industry and year effects.

Another interesting observation from Japanese firm level data is that a significant number of firms stay in the market even if their profits are negative. About 8 percent of Japanese firms in our sample report negative profits in a given year. This fraction becomes even bigger at 11 percent among the firms who always export, the biggest firms. Despite such negative profits, Japanese listed firms do not easily exit from the business, although entry into and exit from the export market are more frequent.

Given these empirical observations, we choose firms to produce multiple products and are heterogeneous in terms of the number of the products as well as the productivity distribution. Firms choose which products to produce and which products to export. Thus Melitz style extensive margin adjustment is mainly at the product level (even though there are endogenous entries and exits of firms). This firm and product level heterogeneity helps explain the weak relationships among size, the export share and profitability in our firm-level data. Our firms also face recurrent idiosyncratic productivity shocks, and thus they may not exit with temporary

percent of all employees (Fukao, et. al., 2008).

negative profits in order to enjoy the option value of continuing production.[6,7] This option value provide a clue for out empirical finding that Japanese firms with negative profits resist to exit from their business.[8]

In Section 2, we present a model of small open economy. The equilibrium dynamics and steady state of the model economy are characterized in Section 3. In Section 4, we calibrate the model. In Section 5, aggregate dynamics is simulated. Section 6 concludes.

2 Model

There is a continuum of home firms $h \in \mathcal{H}_t$. Home firm h produces possibly multiple I_{ht} number of differentiated products for home and export markets at date t. Firm h produces q^H_{hit} amount of the i th differentiated product for the home market using labor l^H_{hit} and imported intermediate input m^{*H}_{hit}, according to a constant returns to scale technology

$$q^H_{hit} = a_{hit} Z_t \left(\frac{l^H_{hit}}{\gamma_L} \right)^{\gamma_L} \left(\frac{m^{*H}_{hit}}{1-\gamma_L} \right)^{1-\gamma_L}, \text{ for } i = 1, 2, .., I_{ht}.$$

A variable a_{hit} is the productivity of firm h to produce the i th differentiated product at date t, Z_t is the aggregate productivity shock, and $\gamma_L \in (0,1)$ is the labor share. We assume no two firms produce the same product and distinguish the differentiated product by (h, i) - the i th product of firm h. Producing a differentiated product for export market has the same marginal productivity with the production for home market, but requires a constant fixed cost ϕ in terms

[6] Ghironi and Melitz (2005) analyze the dynamic effects of an aggregate productivity shock on the real exchange rate in a general equilibrium model with heterogeneous firms. But they concentrate on the extensive margin of products for export. Because there are no further idiosyncratic shocks after entry, there is no endogenous exit nor negative profits in their model.

[7] More broadly, our paper is related to the recent policy literature that examines how much of a real exchange rate depreciation is necessary to close a nation's current account imbalances. Obstfeld and Rogoff (2004) use a three-country model to calculate how much of a depreciation in the real exchange rate is needed to set the U.S. current account to zero. Dekle, Eaton, and Kortum (2008) fit their model to bilateral trade flows for 42 countries and solve for the new equilibrium in real exchange rates to eliminate all current account imbalances.

[8] Strictly speaking, in our sample of Japanese listed firms, firms that drop out of the sample are "delisted." Of the 2386 firms in our sample that we examine between 1985 and 1999, 104 firms became "delisted." We examined the circumstances surrounding the de-listing of all of these 104 firms and the vast majority were delisted because of bankruptcy or "ceasing to do business." A small number disappeared as independent firms because of mergers with stronger firms. Thus, we are on reasonably firm ground when we equate a firm that has been "delisted" as essentially "exiting" from production.

of input composite for each variety as

$$q_{hit}^F = a_{hit} Z_t \left[\left(\frac{l_{hit}^F}{\gamma_L} \right)^{\gamma_L} \left(\frac{m_{hit}^{*F}}{1-\gamma_L} \right)^{1-\gamma_L} - \phi \right], \text{ for } i = 1, 2, ..I_{ht}.$$

Home final goods for home market is produced from all the differentiated products of home market according to a constant returns to scale CES production function as

$$Q_t^H = \left[\int_{h \in \mathcal{H}_t} \left(\sum_{i=1}^{I_{ht}} q_{hit}^{H\ \frac{\theta-1}{\theta}} \right) dh \right]^{\frac{\theta}{\theta-1}},$$

where $\theta > 1$ is the elasticity of substitution between products. Home final goods for export market is produced from the differentiated products of export market as

$$Q_t^F = \left[\int_{h \in \mathcal{H}_t} \left(\sum_{i=1}^{I_{ht}} q_{hit}^{F\ \frac{\theta-1}{\theta}} \right) dh \right]^{\frac{\theta}{\theta-1}}.$$

Any new entrant who pays a sunk cost κ_E in terms of home final goods at date t draws an opportunity of producing b number of new products at date $t+1$, where

$$b = \begin{cases} 2, & \text{with probability } \iota, \\ 1, & \text{with probability } 1-\iota. \end{cases}$$

Thus the average number of new products drawn is equal to $1+\iota$. The productivity $a_{hi,t+1}$ of each product (h,i) is independently and identically distributed such that $a_{hi,t+1} = 0$ with probability $1-\lambda'$, while, with probability λ', $a_{hi,t+1}$ takes a positive value following Pareto distribution with lower bound parameter 1 and the shape parameter α. That is

$$a_{hi,t+1} \begin{cases} = 0, & \text{with probability } 1-\lambda' \\ \in [1, a], & \text{with probability } \lambda' F(a) = \lambda'(1-a^{-\alpha}). \end{cases}$$

The density function of the Pareto distribution is

$$f(a) \equiv F'(a) = \alpha a^{-(\alpha+1)}, \text{ for } a \in [1, \infty).$$

Thus new entrants are heterogeneous in terms of number of products (the "width" b) as well as distribution of productivity (the "height" $a_{hi,t+1}$).

We make two assumptions on the parameters

$$\lambda \equiv (1+\iota)\lambda' < 1, \qquad \text{(Assumption 1)}$$

$$\alpha > 1 \text{ and } \alpha > \theta - 1. \qquad \text{(Assumption 2)}$$

Assumption 1 implies that the average number of products with positive productivity is less than unity per new draw. Assumption 2 says that the shape parameter α of the Pareto distribution is bounded below by one and $\theta - 1$, which later guarantees that CES aggregates of final goods are well behaving.

An incumbent firm who already has existing products must pay fixed maintenance cost κ (in terms of home final goods) for each product in order to produce and maintain its productivity. That is, the firm that wants to maintain I_{ht} number of products must pay κI_{ht}. If the firm does not pay the fixed cost for an existing product, it loses the technology for this product for sure and forever. For the product which the firm pays the maintenance cost, the same productivity is maintained in the next period ($a_{hi,t+1} = a_{hit}$) with probability $1-\delta$, while, with probability δ, it receives a new draw for both width and height productivity according to the same distribution of new entrants. Thus, the number of products each firm produces may increase or decrease depending on the new draw of width and height. Because firms are heterogeneous in the number of products as well as in the productivity distribution, we can show that there are only weak relationships among size, the export share and profitability across firms - an important feature of our Japanese firm-level data.

Home final goods are either consumed by households and government, or used for the entry sunk costs of the new entrants, or for the maintenance costs of the existing technology,

$$Q_t^H = C_t + G_t + \kappa_E N_{Et} + \kappa N_t. \tag{1}$$

Variables C_t and G_t are consumption of households and government, N_{Et} is the measure of entering firms, and N_t is the measure of existing differentiated products which incumbent firms try to maintain. We consider the costs of drawing new technology and maintaining old technology as *intangible* capital investment. We abstract from tangible capital investment.

A representative household supplies labor L_t to earn wage income, consumes final goods C_t, and holds home and foreign real bonds D_t and D_t^* to maximize the expected utility at the initial date $t = 0$

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left(\ln C_t - \psi_0 \frac{L_t^{1+1/\psi}}{1+1/\psi} + \xi_t^* \ln D_t^* \right),$$

subject to the budget constraint

$$C_t + D_t + \epsilon_t D_t^* = w_{Lt} L_t + \Pi_t + R_{t-1} D_{t-1} + \epsilon_t R_{t-1}^* D_{t-1}^* - T_t, \forall t. \tag{2}$$

Variable ϵ_t is real exchange rate (the relative price of foreign to home final goods), w_{Lt} is real wage rate, Π_t is the sum of real net profits distribution of firms, R_{t-1} and R_{t-1}^* are home and foreign one-period real gross interest rates (which are promised at date $t-1$), and T_t is lump-sum tax. Note that, although both home and foreign bonds are used as means of saving, we assume that the holding of foreign bonds facilitates international transactions, hence is in the utility function. The utility from holding foreign bonds is subject to the "liquidity shock" ξ_t^*.[9]

We assume that all home imports are intermediate inputs to production, and that the imported input price is normalized to be one in terms of foreign final goods. We assume that foreign aggregate demand for home exports are given by

$$Q_t^F = \left(p_t^F\right)^{-\varphi} Y_t^*, \tag{3}$$

where Y_t^* is an exogenous foreign demand parameter and p_t^F is an endogenous export price in terms of foreign final goods. A parameter φ is the elasticity of demand for home export final goods, which we assume it to be relatively inelastic

$$0 < \varphi < 1. \tag{4}$$

We assume that foreigners do not hold home bond. Then, foreign bond holdings D_t^* of the home household evolves along with exports and imports as

$$D_t^* = R_{t-1}^* D_{t-1}^* + p_t^F Q_t^F - M_t^*, \tag{5}$$

where $M_t^* = \int_{h \in \mathcal{H}_t} \left[\sum_{i=1}^{I_{ht}} (m_{hit}^{*H} + m_{hit}^{*F})\right] dh$ is the total imported input of the home country.

The government budget constraint is given by

$$D_t = R_{t-1} D_{t-1} + G_t - T_t. \tag{6}$$

[9] The idea is similar to money in the utility function. Section 5.3.8 of Obstsfeld and Rogoff (1998) presents a model with both home and foreign money in the utility function to analyze the phenomenon of dollarization. Alternatively, we can formulate that home households face an international borrowing constraint and that the utility from foreign bond holding is $\xi^* \ln(D_t^* + \zeta_t^*)$ where $\zeta_t^* > 0$ is the credit line of foreign lenders to the home representative household which is stochastic. We ignore the utility of home bonds for simplicity.

Here, because the foreigners do not hold home bond, the home bond holding of the representative household is equal to the government bond supply. For a given autonomous government expenditure G_t, government adjusts tax T_t to stabilize the outstanding debt such that

$$T_t - T = \zeta_T \left[R_{t-1} D_{t-1} - RD \right], \tag{7}$$

where T and RD are the steady-state values of tax and government debt at the beginning of period.

3 Competitive Equilibrium

3.1 Firm's Production

The market for final goods and factors of production are perfectly competitive, while the market for differentiated products are monopolistically competitive. From the usual feature of the CES production function of final goods from differentiated products, each firm faces a downward sloping demand curve for the product variety in home and foreign markets as a function of its prices p_{hit}^H and p_{hit}^F, such that

$$\begin{aligned} q_{hit}^H &= \left(\frac{p_{hit}^H}{p_t^H} \right)^{-\theta} Q_t^H, \\ q_{hit}^F &= \left(\frac{p_{hit}^F}{p_t^F} \right)^{-\theta} Q_t^F, \end{aligned}$$

where p_t^H and p_t^F are the aggregate price indices of final goods in home and export markets given by

$$\begin{aligned} p_t^H &= \left[\int_{h \in \mathcal{H}_t} \left(\sum_{i=1}^{I_{ht}} \left(p_{hit}^H \right)^{1-\theta} \right) dh \right]^{\frac{1}{1-\theta}} = 1, \\ p_t^F &= \left[\int_{h \in \mathcal{H}_t} \left(\sum_{i=1}^{I_{ht}} \left(p_{hit}^F \right)^{1-\theta} \right) dh \right]^{\frac{1}{1-\theta}}. \end{aligned} \tag{8}$$

We use home final goods as the numeraire in the home market (i.e., $p_t^H = 1$), and foreign final goods as the numeraire in the foreign market.

Recall that the production function of differentiated products all have a common component: Cobb-Douglas function of input composite of labor and imported intermediate input.

Moreover, the ratios of labor to imported intermediate input are equal across firms when firms minimize the costs under perfectly competitive factor market. Let x_{hit}^H and x_{hit}^F be input composites used for producing differentiated products for the home and export markets. Then the production function can be simplified to

$$\begin{aligned} q_{hit}^H &= a_{hit} Z_t \cdot x_{hit}^H, \\ q_{hit}^F &= a_{hit} Z_t \cdot \left(x_{hit}^F - \phi \right). \end{aligned}$$

Then, the sum of input composite use is equal to the aggregate production of the input composite,

$$\int_{h \in \mathcal{H}_t} \left(\sum_{i=1}^{I_{ht}} (x_{hit}^H + x_{hit}^F) \right) dh \equiv X_t = \left(\frac{L_t}{\gamma_L} \right)^{\gamma_L} \left(\frac{M_t^*}{1-\gamma_L} \right)^{1-\gamma_L}.$$

Because the price of imported inputs at home is equal to the real exchange rate (due to our choice of numeraire), the cost minimization implies that the unit cost of the input composite w_t and the demands for labor and imported inputs are given by

$$w_t = (w_{Lt})^{\gamma_L} \epsilon_t^{1-\gamma_L}, \tag{9}$$

$$L_t = \gamma_L \frac{w_t X_t}{w_{Lt}}, \tag{10}$$

$$M_t^* = (1-\gamma_L) \frac{w_t X_t}{\epsilon_t}. \tag{11}$$

Maximizing current profits, each firm sets the product prices p_{hit}^H and p_{hit}^F as mark-ups over their unit production cost such that

$$p_{hit}^H = \frac{\theta}{\theta-1} \frac{w_t}{a_{hit} Z_t} \equiv p_t^H(a_{hit}), \tag{12}$$

$$p_{hit}^F = \frac{\theta}{\theta-1} \frac{w_t/\epsilon_t}{a_{hit} Z_t} \equiv p_t^F(a_{hit}). \tag{13}$$

Then, the quantities q_{hit}^H and q_{hit}^F of each product for home and foreign market depend on its own height productivity a_{hit} only (aside from aggregate variables) such that

$$q_{hit}^H = \left(\frac{p_t^H(a_{hit})}{p_t^H} \right)^{-\theta} Q_t^H \equiv q_t^H(a_{hit}), \tag{14}$$

$$q_{hit}^F = \left(\frac{p_t^F(a_{hit})}{p_t^F} \right)^{-\theta} Q_t^F \equiv q_t^F(a_{hit}). \tag{15}$$

That is, although each firm may produce multiple differentiated products, firm's choice on how much to produce and whether to continue to produce for each product is independent from the choices of other products, like the "amoeba management".[10]

We conjecture that in equilibrium, all firms choose to pay the fixed maintenance cost for the product with positive productivity (which we will confirm later). Then, the total measure of differentiated products evolves through maintenance and new entries as:

$$N_{t+1} = (1 - \delta + \delta\lambda)\, N_t + \lambda N_{Et}. \tag{16}$$

The first term in the right hand side is the measure of maintained products in which $1-\delta+\delta\lambda < 1$ by Assumption 1. Let $N_t(a)$ be the measure of products with productivity a. Then, from the specific feature of our idiosyncratic productivity evolution, $N_t(a)$ is a proportional to N_t as:

$$N_t(a) = f(a) N_t.$$

Thus, from (8) and (12), the price index for home final goods for the home market becomes

$$1 = p_t^H = \left[\int_1^\infty p_t^H(a)^{1-\theta} N_t f(a) da\right]^{\frac{1}{1-\theta}} = \frac{\theta}{\theta - 1}\frac{w_t}{A_t^H}.$$

Variable A_t^H is the aggregate productivity of home firms in home market, given by

$$A_t^H \equiv \overline{a} N_t{}^{\frac{1}{\theta-1}} Z_t, \tag{17}$$

and $\overline{a}$ is the average productivity of products that are produced for home market,

$$\overline{a} \equiv \left[\int_1^\infty a^{\theta-1} f(a) da\right]^{\frac{1}{\theta-1}} = \left(\frac{\alpha}{\alpha + 1 - \theta}\right)^{\frac{1}{\theta-1}}.$$

Note that this implies that the unit cost of input composite is given by

$$w_t = \frac{\theta - 1}{\theta} A_t^H. \tag{18}$$

[10] The founder of Kyocera (a Japanese technology company), Mr. Kazuo Inamori, proposes an "amoeba" management style, in which each production unit makes relatively independent production decisions, while the number of production units multiply and shrink like "amoebas." Our technology can be seen as a justification for the "amoeba" management style. See also Bernard, Redding and Schott. (2010, 2011).

Due to the presence of the fixed cost of exporting, we conjecture that there is a lower bound of productivity level $\underline{a}_t > 1$ at which the product makes zero profit for exporting such that

$$\pi_t^F\left(\underline{a}_t\right) = \epsilon_t p_t^F(\underline{a}_t) q_t^F\left(\underline{a}_t\right) - w_t\left[\frac{q_t^F\left(\underline{a}_t\right)}{\underline{a}_t Z_t} + \phi\right] = w_t\left[\frac{1}{\theta-1}\frac{q_t^F\left(\underline{a}_t\right)}{\underline{a}_t Z_t} - \phi\right] = 0, \tag{19}$$

Thus only a fraction $Prob(a \geq \underline{a}_t) = \left(\underline{a}_t\right)^{-\alpha} < 1$ of maintained products are exported.

In Appendix A, we show that the lower bound of productivity for export which clears the export market is given by

$$\underline{a}_t = \left[\frac{\alpha\left(\theta-1\right)\phi}{\alpha+1-\theta}\frac{A_t^H N_t}{\epsilon_t^{\varphi} Y_t^*}\right]^{\frac{\theta-1}{\alpha(\theta-1)+(\alpha+1-\theta)(1-\varphi)}}. \tag{20}$$

(The details of the competitive equilibrium are all in Appendix A.) We verify the conjecture that $\underline{a}_t > 1$ so that some products with low productivity are not exported, if and only if

$$\frac{\epsilon_t^{\varphi} Y_t^*}{A_t^H N_t} < \frac{\alpha\left(\theta-1\right)\phi}{\alpha+1-\theta}. \tag{Condition 1}$$

If this condition is not satisfied, all home products would be exported, which is at odd with the data. Thus, we restrict our attention to the case where Condition 1 is satisfied.

The export sales S_t^F in terms of home final good turns out to be

$$\begin{aligned} S_t^F &\equiv \epsilon_t p_t^F Q_t^F \\ &= \left(\underline{a}_t\right)^{\frac{(\alpha+1-\theta)(1-\varphi)}{\theta-1}} \epsilon_t^{\varphi} Y_t^*. \end{aligned} \tag{21}$$

3.2 Market Clearing and Free Entry

From the utility maximization of the representative household, we have

$$1 = R_t E_t\left(\Lambda_{t,t+1}\right), \tag{22}$$

$$\xi_t^* \frac{C_t}{D_t^*} = \epsilon_t - R_t^* E_t\left(\Lambda_{t,t+1}\epsilon_{t+1}\right), \tag{23}$$

$$w_{Lt} = \psi_0 L_t^{\frac{1}{\psi}} C_t, \tag{24}$$

where $\Lambda_{t,t+1} = \beta C_t / C_{t+1}$. The first equation is a standard Euler equation for home bond holding. The second equation is an Euler equation for foreign bond holding, where the left hand side is the marginal rate of substitution between foreign bond holdings and consumption and the right hand side term is the opportunity cost of holding one unit of the foreign bond for one period. The third equation is the labor supply condition.

We show in Appendix that the market clearing condition of labor and input composite implies

$$X_t = \frac{1}{\gamma_L(\psi_0 C_t)^{\psi}} \left[\frac{{w_t}^{1-\gamma_L+\psi}}{\epsilon_t^{(1-\gamma_L)(1+\psi)}} \right]^{\frac{1}{\gamma_L}} \tag{25}$$

$$= X_t^H + \phi \frac{\theta\alpha + 1 - \theta}{\alpha + 1 - \theta} \left(\underline{a}_t\right)^{-\alpha} N_t. \tag{26}$$

where X_t^H denote the aggregate composite input use for the home market. The home final goods market clearing implies

$$C_t + G_t + \kappa_E N_{Et} + \kappa N_t = A_t^H X_t^H. \tag{27}$$

From (5), (11) and (21), foreign bond holding evolves as

$$D_t^* = R_{t-1}^* D_{t-1}^* + \left(\underline{a}_t\right)^{\frac{(\alpha+1-\theta)(1-\varphi)}{\theta-1}} \epsilon_t^{\varphi-1} Y_t^* - (1-\gamma_L)\frac{w_t X_t}{\epsilon_t} \tag{28}$$

Let $V_t(a)$ be the value of the product with productivity a at the beginning of period (for which the fixed cost of maintenance is paid). The Bellman equation is

$$\begin{aligned} V_t(a) &= \pi_t^H(a) + \pi_t^F(a) - \kappa \\ &\quad + E_t \Lambda_{t,t+1} \left[(1-\delta) V_{t+1}(a) + \delta\lambda \int_1^{\infty} V_{t+1}(a') f(a') da' \right], \end{aligned}$$

where $\pi_t^H(a)$ and $\pi_t^F(a)$ are profit arising from selling a product with productivity $a_{hit} = a$ in the home and export markets. The free entry condition for a potential entrant is

$$\kappa_E = \lambda E_t \left(\Lambda_{t,t+1} \overline{V}_{t+1} \right), \tag{29}$$

where $\overline{V}_t$ is the average value of the products produced as

$$\begin{aligned} \overline{V}_t &\equiv \int_1^{\infty} V_t(a) f(a) da \\ &= \overline{\pi}_t - \kappa + (1-\delta+\delta\lambda) E_t(\Lambda_{t,t+1} \overline{V}_{t+1}), \end{aligned} \tag{30}$$

and $\overline{\pi}_t$ is the average profit of the products with positive productivity $\overline{\pi}_t \equiv \int_1^\infty \left\{\pi_t^H(a) + \pi_t^F(a)\right\} f(a)da$.

In Appendix, we show that the free entry condition can be written as

$$\kappa_E \left[1 - (1 - \delta + \delta\lambda) E_t \left(\Lambda_{t,t+1}\right)\right] = \lambda E_t[\Lambda_{t,t+1}(\overline{\pi}_{t+1} - \kappa)]. \tag{31}$$

The left-hand side is the cost of increasing entry by one unit now and reducing entry by $1-\delta+\delta\lambda$ in the next period. This increases the expected number of products with positive productivity by λ only in the next period. The right-hand side is the expected increase of the net profit in the next period. We can also show the average profit is

$$\overline{\pi}_t = \frac{w_t X_t}{(\theta - 1)N_t} - \frac{\theta}{\theta - 1} w_t \phi \cdot \left(\underline{a}_t\right)^{-\alpha}. \tag{32}$$

The first term in the right hand side is the average profit due to mark-up per product and the second term is the average fixed cost for export.

The necessary and sufficient condition that the firm strictly prefers to maintain a product with the lowest productivity by paying the fixed cost is $V_t(1) > 0$ for all t. A sufficient condition for this is

$$0 < \pi_t^H(1) - \kappa + \delta\kappa_E, \ \forall t. \tag{Condition 2}$$

Notice that this condition is satisfied even if realized current net profits of each product is negative ($\pi_t^H(1) < \kappa$), because there is an option value for the low productivity product to become a high productivity product. This helps explain why firms often record negative profits after paying their fixed costs of maintaining the business. In addition, because firms may have a large number of low productivity products, there can be only a weak correlation between size and profitability across firms - another interesting aspect of Japanese firms.

The option value of improving idiosyncratic productivity cannot be too large, because we conjecture that the firm will not maintain a product with zero productivity. The condition for the firm not to maintain an unproductive product is

$$-\kappa + \delta\kappa_E < 0. \tag{Assumption 4}$$

3.3 Equilibrium Dynamics

The state of our economy is described by the set of variables $\mathcal{M}_t = (N_t, D_{t-1}, D_t^*, Z_t, G_t, Y_t^*, \xi_t^*, R_t^*)$ where the first three state variables are endogenous and the last five are exogenous. The equilibrium dynamics of our economy is described by the fourteen endogenous variables of $(A_t^H, w_t, \underline{a}_t, X_t, X_t^H, C_t, R_t, T_t, D_t, \overline{\pi}_t, N_{Et}, N_{t+1}, \epsilon_t, D_t^*)$ as functions of $\mathcal{M}_t$ which are determined by the fourteen equations: (i) government behavior (6) and (7), and (ii) market equilibrium conditions (16), (17), (18), (20), (22), (23), (25), (26), (27), (28), (31) and (32). The consumer budget constraint (2) is automatically satisfied once all the market clearing conditions are satisfied (by a variant of Walras' Law), noting that aggregate net profit distribution is equal to the average gross profit multiplied by the number of products produced net of intangible investment cost ($\Pi_t = \overline{\pi}_t N_t - \kappa N_t - \kappa_E N_{Et}$).

We can organize the equilibrium conditions. Aggregate productivity A_t^H and unit cost of input composite w_t are functions of only state variables . Given A_t^H and w_t, the variables $\left(\underline{a}_t, X_t, X_t^H, \overline{\pi}_t, N_{Et}\right)$ can be arranged into functions of $(C_t, \epsilon_t, N_{t+1})$ and the state variables. Once we find R_t as a function of (C_t, C_{t+1}), the variables (T_t, D_t) are determined by the government budget constraint and the fiscal rule, independent from other variables. Thus, the equilibrium dynamics are characterized by four variables $(C_t, \epsilon_t, D_t^*, N_{t+1})$ as a function of $(N_t, D_{t-1}^*, Z_t, G_t, Y_t^*, \xi_t^*, R_t^*)$ that satisfies the following four dynamic equations:

(i) Euler equation for foreign bond holding

$$\xi_t^* \frac{C_t}{D_t^*} = \epsilon_t - R_t^* E_t \left(\beta \frac{C_t}{C_{t+1}} \epsilon_{t+1} \right); \tag{33}$$

(ii) Dynamics of net foreign asset: (28);

(iii) Free entry equation, obtained from combining equations (31) and (32)

$$\begin{aligned} & \kappa_E \left[1 - (1 - \delta + \delta\lambda) E_t \left(\beta \frac{C_t}{C_{t+1}} \right) \right] \\ = \; & \lambda E_t \left(\beta \frac{C_t}{C_{t+1}} \left\{ -\kappa + A_{t+1}^H \left[\frac{1}{\theta} \frac{X_{t+1}}{N_{t+1}} - \phi(\underline{a}_{t+1})^{-\alpha} \right] \right\} \right), \end{aligned} \tag{34}$$

where A_{t+1}^H, $\underline{a}_{t+1}$, and X_{t+1} are functions of N_{t+1}, ϵ_{t+1}, C_{t+1} and exogenous variables;

(iv) Home final goods market clearing condition,

$$\begin{aligned} & C_t + G_t + \frac{\kappa_E}{\lambda}\left[N_{t+1} - (1-\delta+\delta\lambda)N_t\right] + \kappa N_t \\ = \quad & A_t^H \left[X_t - \frac{\alpha\theta + 1 - \theta}{\alpha + 1 - \theta}\phi(\underline{a}_t)^{-\alpha}\right]; \end{aligned} \tag{35}$$

After characterizing the equilibrium, we verify that conditions (Condition 1) and (Condition 2) are satisfied in equilibrium.

Appendix B derives the steady state of this economy.

3.4 Dynamics of "Shrunk" Model

In order to examine the dynamics of our model, we first examine the market clearing condition for net foreign assets (33). Suppose, as it is likely, that a liquidity shock to foreign bonds ξ_t^* is very volatile in the short-run. The supply of net foreign assets changes sluggishly over time through the current account. Consumption is relatively smooth by permanent income theory if the investment on intangible capital serves as a buffer to absorb shocks (which we will verify later). Then, since the liquidity shock to foreign bonds appears only in the market clearing condition for net foreign assets, the real exchange rate has to adjust quickly to the volatile movement of the liquidity shock at high frequency - even though at low frequency, the adjustment of the current account and consumption are as important as the real exchange rate adjustment. That is, in our economy, the high frequency movement of the real exchange rate is dominated by the liquidity shock. Thus, we can treat the short-run movement of the real exchange rate as almost "exogenous," because we can always find a liquidity shock to justify the observed movement of real exchange rates as long as our boundary conditions (Condition 1) and (Condition 2) are satisfied and the evolution of net foreign assets is stable in the long-run. In addition, net foreign assets appear only in the equation for the evolution of net foreign assets (28).

Therefore, we first consider a "shrunk" model, taking real exchange rate as exogenous: We consider $\mathcal{M}_t' = (N_t, Z_t, G_t, Y_t^*, \epsilon_t)$ as state variables, in which N_t is endogenous and the others are exogenous. Given the explicit expression of $\left(A_t^H, \underline{a}_t, X_t\right)$ as a function of the state variables

$\mathcal{M}_t'$ in $(17, 20, 25, 18)$, the dynamic equilibrium is characterized by (C_t, N_{t+1}) which satisfies $(34, 35)$ as a function of $\mathcal{M}_t'$.

3.5 Exchange Rate Disconnect Puzzle

Recall that the lower bound of the productivity for export is

$$\underline{a}_t = \left[\frac{\alpha(\theta - 1)\phi}{\alpha + 1 - \theta} \frac{A_t^H N_t}{\epsilon_t^{\varphi} Y_t^*} \right]^{\frac{\theta - 1}{\alpha(\theta-1) + (\alpha+1-\theta)(1-\varphi)}}.$$

Because the measure of products N_t is a state variable, the extensive margin $\underline{a}_t$ reacts to shifts in the real exchange rate, foreign demand and domestic productivity contemporaneously. When the elasticity of foreign demand for home products (φ) is relatively small, the lower bound of productivity for export is relatively insensitive to the real exchange rate shift.

From $(13, 15, 19, 21)$, the real value of export at product level and the aggregate are

$$\begin{aligned} s_t^F(a) &\equiv \epsilon_t p_t^F(a) q_t^F(a) = (\theta - 1)\phi A_t^H \left(\frac{a}{\underline{a}_t} \right)^{\theta - 1} I(a - \underline{a}_t), \\ S_t^F &= \epsilon_t p_t^F Q_t^F = \left\{ \left[\frac{\alpha(\theta-1)\phi}{\alpha + 1 - \theta} A_t^H N_t \right]^{(\alpha+1-\theta)(1-\varphi)} (\epsilon_t^{\varphi} Y_t^*)^{\alpha(\theta-1)} \right\}^{\frac{1}{\alpha(\theta-1)+(\alpha+1-\theta)(1-\varphi)}}, \end{aligned}$$

where $I(a - \underline{a}_t)$ is an indicator function such that $I(a - \underline{a}_t) = 1$ if $a - \underline{a}_t \geq 0$, and $= 0$ otherwise. The response of the export value of an individual product to the real exchange rate depends upon whether there is an adjustment of the extensive margin. If a product has a very high productivity and is always exported (as always $a > \underline{a}_t$), then the export value of such product is not very responsive to the real exchange rate is small because the lower bound for the export is not very sensitive. Figure 2a describes the relationship between the export value of a high productive product and the real exchange rate. If a product has a productivity in the neighborhood of the lower bound for the export, then the response of the export value is large because both intensive and extensive margins adjust to the real exchange rate. Figure 2b describes the response of the export value of a marginal product. When the real exchange rate appreciates (ϵ_t falls), the lower bound of productivity for export increases. At some threshold ϵ^*, the productivity of this product becomes lower than the boundary, and the export value drops to zero. As in Green (2009), the exports of the low productivity products drop like "flies" when there is an adverse shock such as a real exchange rate appreciation.

Our Japanese firm-level data (Kaigin data) are mostly of relatively large firms, which typically produce multiple products - possibly after a number of new draws of $b = 2$. If a majority of products of some firm is close to the lower bound for export, then the export of this firm is sensitive to the real exchange rate shifts as in Figure 2b. Because such firms are common under Assumption 2, the firm-level export tends to react significantly to the real exchange rate. In contrast, the products with considerably higher productivity than the lower bound is not very sensitive to the real exchange rate shifts as in Figure 2a, and their share in the aggregate export is large. Thus the aggregate exports are less sensitive contemporaneously to the real exchange rate shift as in Figure 2c. This heterogeneous reaction of exports to the real exchange rate shift across different products explains why firm level exports co-move significantly with the real exchange rate, while aggregate exports appear "disconnected" from the real exchange rate.[11]

4 Calibration

5 Simulation

6 Conclusion

[11] Our explanation of the extensive margin adjustment at product level is consistent with Dekle, Jeong and Ryoo (2007), which find that the apparent lack of relationship between the exchange rate and aggregate exports occur through the intensive margin of export sales within firms, rather than through the extensive margin of entry and exit of firms in the export market.

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A Details of Competitive Equilibrium

Aggregating the product prices for the export market in (13), the aggregate price index of home final goods for the foreign market is

$$p_t^F = \frac{\theta}{\theta-1}\frac{w_t/\epsilon_t}{\overline{a}_t^F N_t^{\frac{1}{\theta-1}} Z_t} = \frac{1}{\epsilon_t}\frac{\overline{a}}{\overline{a}_t^F}, \tag{36}$$

where $\overline{a}_t^F$ is the average productivity of the exported products given by

$$\overline{a}_t^F \equiv \left[\int_{\underline{a}_t}^{\infty} a^{\theta-1} f(a) da\right]^{\frac{1}{\theta-1}} = \left[\frac{\alpha}{\alpha+1-\theta}\left(\underline{a}_t\right)^{\theta-\alpha-1}\right]^{\frac{1}{\theta-1}} = \overline{a}\cdot\left(\underline{a}_t\right)^{-\frac{\alpha+1-\theta}{\theta-1}}.$$

The zero profit condition for export implies

$$q_t^F\left(\underline{a}_t\right) = (\theta-1)\,\phi \underline{a}_t Z_t.$$

Using the property $q_t^F(a)/q_t^F\left(\underline{a}_t\right) = \left(p_t^F(a)/p_t^F\left(\underline{a}_t\right)\right)^{-\theta} = \left(a/\underline{a}_t\right)^{\theta}$ for $a > \underline{a}_t$ from (13) and (15), we have the aggregate supply of home export as

$$\begin{aligned} Q_t^F &= \left[\int_{\underline{a}_t}^{\infty} q_t^F(a)^{\frac{\theta-1}{\theta}} f(a) N_t da\right]^{\frac{\theta}{\theta-1}} \\ &= q_t^F\left(\underline{a}_t\right) N_t^{\frac{\theta}{\theta-1}}\left[\int_{\underline{a}_t}^{\infty}\left(\frac{a}{\underline{a}_t}\right)^{\theta-1} f(a) da\right]^{\frac{\theta}{\theta-1}} \\ &= (\theta-1)\,\phi Z_t N_t^{\frac{\theta}{\theta-1}} \overline{a}^{\theta}\left(\underline{a}_t\right)^{-\frac{\alpha\theta+1-\theta}{\theta-1}}. \end{aligned}$$

Substituting the export price index p_t^F in (36) into the the export demand equation in (3), the aggregate demand for export is given by

$$Q_t^F = \left(\underline{a}_t\right)^{-\frac{\alpha+1-\theta}{\theta-1}\varphi} \epsilon_t^{\varphi} Y_t^*. \tag{37}$$

Then, the export market clearing condition solves for the cutoff productivity $\underline{a}_t$ such that

$$\begin{aligned} \underline{a}_t &= \left[\frac{(\theta-1)\,\phi\overline{a}^{\theta} Z_t N_t^{\frac{\theta}{\theta-1}}}{\epsilon_t^{\varphi} Y_t^*}\right]^{\frac{\theta-1}{\alpha(\theta-1)+(\alpha+1-\theta)(1-\varphi)}} \\ &= \left[\frac{\alpha\,(\theta-1)\,\phi}{\alpha+1-\theta}\frac{A_t^H N_t}{\epsilon_t^{\varphi} Y_t^*}\right]^{\frac{\theta-1}{\alpha(\theta-1)+(\alpha+1-\theta)(1-\varphi)}}. \end{aligned}$$

This is (20) in the text.

From $(36, 37)$, the home export value in terms of home final goods is

$$S_t^F \equiv \epsilon_t p_t^F Q_t^F = (\underline{a}_t)^{\frac{(\alpha+1-\theta)(1-\varphi)}{\theta-1}} \epsilon_t^{\varphi} Y_t^*.$$

This is (21) in the text.

The labor supply condition (24)together with the composite input price equation (9) can be written as

$$L_t = \frac{1}{(\psi_0 C_t)^{\psi}} \left(\frac{w_t}{\epsilon_t^{1-\gamma_L}} \right)^{\frac{\psi}{\gamma_L}}.$$

Similarly, the labor demand equation (10) together with the equation (9) can be written as

$$X_t = \left(\frac{w_t}{\epsilon_t} \right)^{\frac{1-\gamma_L}{\gamma_L}} \frac{L_t}{\gamma_L}.$$

Then from labor market clearing condition), we have the aggregate composite input as

$$\begin{aligned} X_t &= \frac{1}{\gamma_L (\psi_0 C_t)^{\psi}} \left[\frac{{w_t}^{1-\gamma_L+\psi}}{\epsilon_t^{(1-\gamma_L)(1+\psi)}} \right]^{\frac{1}{\gamma_L}} \\ &= X_t^F + X_t^H, \end{aligned}$$

where X_t^F and X_t^H denote the aggregate composite input use for export market and for the home market. Using $(12, 12, 14, 15)$, we have

$$\begin{aligned} X_t^F &= \int_{\underline{a}_t}^{\infty} \left[\frac{q_t^F(a)}{a Z_t} + \phi \right] f(a) N_t da \\ &= \int_{\underline{a}_t}^{\infty} \phi \left[\left(\frac{a}{\underline{a}_t} \right)^{\theta-1} (\theta - 1) + 1 \right] f(a) N_t da \\ &= \phi \frac{\theta\alpha + 1 - \theta}{\alpha + 1 - \theta} (\underline{a}_t)^{-\alpha} N_t, \end{aligned}$$

$$\begin{aligned} X_t^H &= \int_1^{\infty} \frac{q_t^H(a)}{a Z_t} f(a) N_t da \\ &= \frac{q_t^H(1)}{Z_t} \int_1^{\infty} a^{\theta-1} f(a) N_t da \\ &= \frac{Q_t^H}{A_t^H}. \end{aligned}$$

Together with (1), we have $(25, 26, 27)$ in the text.

The profit arising from selling a product with productivity $a_{hit} = a$ in the home market is

$$\begin{aligned}\pi_t^H(a) &\equiv p_t^H(a)q_t^H(a) - w_t x_t^H(a), \\ &= \frac{1}{\theta-1} w_t x_t^H(a).\end{aligned}$$

The profits from exporting a product with productivity $a_{hit} = a \geq \underline{a}_t$ to foreign market is

$$\begin{aligned}\pi_t^F(a) &\equiv \epsilon_t p_t^F(a)q_t^F(a) - w_t x_t^F(a), \\ &= w_t \left[\frac{1}{\theta-1} x_t^F(a) - \frac{\theta}{\theta-1}\phi\right].\end{aligned}$$

Thus we have the average profit as

$$\begin{aligned}\overline{\pi}_t &= \int_1^\infty \left\{\pi_t^H(a) + \pi_t^F(a)\right\} f(a)da \\ &= w_t \left[\frac{X_t}{(\theta-1)N_t} - \frac{\theta}{\theta-1}\phi \cdot (\underline{a}_t)^{-\alpha}\right].\end{aligned}$$

This is (32) in the text.

Combining the free entry condition and the average value function in (29, 30), we have

$$\overline{V}_t = \overline{\pi}_t - \kappa + (1-\delta+\delta\lambda)\frac{\kappa_E}{\lambda}.$$

Substituting this of date t+1 into (29), we we have (31) in the text.

The necessary and sufficient condition that the firm strictly prefers to maintain a product with the lowest productivity by paying the fixed cost is

$$\begin{aligned}0 &< V_t(1) = \pi_t^H(1) - \kappa + E_t\{\Lambda_{t,t+1}\left[(1-\delta)V_{t+1}(1) + \delta\lambda\overline{V}_{t+1}\right]\} \\ &= \pi_t^H(1) - \kappa + \delta\kappa_E + (1-\delta)E_t[\Lambda_{t,t+1}V_{t+1}(1)], \text{ for all } t\end{aligned}$$

Thus a sufficient condition is $(Condition 2)$ in the text. The necessary and sufficient condition that the firm does not maintain a product with zero productivity is

$$\begin{aligned}0 &> V_t(0) = -\kappa + E_t\{\Lambda_{t,t+1}\left[(1-\delta)V_{t+1}(0) + \delta\lambda\overline{V}_{t+1}\right]\} \\ &= -\kappa + \delta\kappa_E + (1-\delta)E_t[\Lambda_{t,t+1}V_{t+1}(0)], \text{ for all } t\end{aligned}$$

Thus the necessary and sufficient condition is $(Assumption 4)$ in the text.

B Steady State

In steady state, $\Lambda_{t,t+1} = \beta$ and $R_t = 1/\beta$. The free entry condition (29) and the average value function in (30) imply that the steady state average profit is given by the following constant:

$$\overline{\pi} = \kappa + \frac{\kappa_E}{\lambda}\left(\frac{1}{\beta} - 1 + \delta - \delta\lambda\right). \tag{38}$$

Or directly from the average profit equation (32), combined with the exporting product fraction equation (**??**), the average profit is related with other equilibrium aggregates such that

$$\overline{\pi} = \frac{w}{\theta - 1}\left[\frac{X}{N} - \theta\phi\underline{a}^{-\alpha}\right]. \tag{39}$$

From the foreign bond holding equation (23), we have

$$\epsilon D_t^* = \frac{\xi_t^* C}{1 - \beta R^*}, \tag{40}$$

which, combined with the current account balance equation (5) together with (11) and (21), implies

$$(1-\gamma_L)\frac{wX}{N} = \underline{a}^{\frac{(\alpha+1-\theta)(1-\varphi)}{\theta-1}}\frac{\epsilon^{\varphi}Y^*}{N} + \frac{\xi_t^*\left(R^*-1\right)}{\left(1-\beta R^*\right)}\frac{C}{N}. \tag{41}$$

The export cut-off productivity equation (20), combined with (18), implies

$$\underline{a}^{\alpha + \frac{(\alpha+1-\theta)(1-\varphi)}{\theta-1}} = \frac{\alpha\phi\theta}{\alpha+1-\theta}\frac{wN}{\epsilon^{\varphi}Y^*}. \tag{42}$$

Combining the above two equilibrium relationships (41) and (42) regarding foreign assets and export markets with the average profit equation (39), we have

$$\begin{aligned}\frac{\alpha\phi\theta w\underline{a}^{-\alpha}}{\alpha+1-\theta} &= \underline{a}^{\frac{(\alpha+1-\theta)(1-\varphi)}{\theta-1}}\frac{\epsilon^{\varphi}Y^*}{N} \\ &= (1-\gamma_L)\frac{wX}{N} - \frac{\xi_t^*\left(R^*-1\right)}{\left(1-\beta R^*\right)}\frac{C}{N} \\ &= \frac{\alpha}{\alpha+1-\theta}\left[\frac{wX}{N} - (\theta-1)\,\overline{\pi}\right],\end{aligned} \tag{43}$$

which can be rearranged into

$$\left(\frac{\theta-1}{\alpha+1-\theta} + \gamma_L\right)\frac{wX}{N} + \frac{\xi_t^*\left(R^*-1\right)}{\left(1-\beta R^*\right)}\frac{C}{N} = \frac{\alpha(\theta-1)}{\alpha+1-\theta}\overline{\pi}. \tag{44}$$

From the total measure of products evolution equation (16),

$$\frac{N_E}{N} = \frac{\delta}{\lambda}(1-\lambda). \tag{45}$$

The final goods market clearing condition (35), we get

$$\frac{C}{N} + \frac{G}{N} + \kappa_E \frac{\delta}{\lambda}(1-\lambda) + \kappa = A^H \left(\frac{X}{N} - \phi \underline{a}^{-\alpha} \frac{\theta\alpha + 1 - \theta}{\alpha + 1 - \theta} \right),$$

which, using (18), (38), and (39), implies

$$\frac{\theta - 1}{\alpha + 1 - \theta} \frac{wX}{N} + \frac{C}{N} = \frac{\alpha(\theta - 1)}{\alpha + 1 - \theta} \overline{\pi} + \frac{\kappa_E}{\lambda} \left(\frac{1}{\beta} - 1 \right) - G/N. \tag{46}$$

Using these equilibrium conditions (44) and (46) for current account and domestic final goods market, we can solve for $\frac{C}{N}$ and $\frac{wX}{N}$ simultaneously as functions of parameters and exogenous variables, conditional on N such that

$$\begin{aligned} \frac{C}{N} &= c(N), \\ \frac{wX}{N} &= wx(N). \end{aligned}$$

From the composite input price equation (18) together with (17) and (20), we have

$$w = \frac{\theta - 1}{\theta} \left(\frac{\alpha}{\alpha + 1 - \theta} \right)^{\frac{1}{\theta - 1}} Z N^{\frac{1}{\theta - 1}} \equiv w(N). \tag{47}$$

Given $c(N)$ and $wx(N)$, combining the equilibrium aggregate quantity of composite input in (25) with (47), the real exchange rate is

$$\epsilon = \left\{ \frac{w}{N^{\gamma_L}} \left[\gamma_L \left(\psi_0 c(N) \right)^{\psi} wx(N) \right]^{-\frac{\gamma_L}{1+\psi}} \right\}^{\frac{1}{1-\gamma_L}} \equiv \epsilon(N). \tag{48}$$

Given $w(N)$ and $\epsilon(N)$, from (20), the export cutoff productivity is

$$\underline{a} = \left[\frac{\alpha\theta\phi}{\alpha + 1 - \theta} \frac{w(N) N}{\epsilon(N)^{\varphi} Y^*} \right]^{\frac{\theta - 1}{\alpha(\theta - \varphi) - (\theta - 1)(1 - \varphi)}} \equiv \underline{a}(N). \tag{49}$$

Given $c(N)$ and $\epsilon(N)$, the steady state foreign bond holding can be found from (40) such that

$$D_t^* = \frac{\xi_t^*}{1 - \beta R^*} \frac{c(N) N}{\epsilon(N)} \equiv D_t^*(N). \tag{50}$$

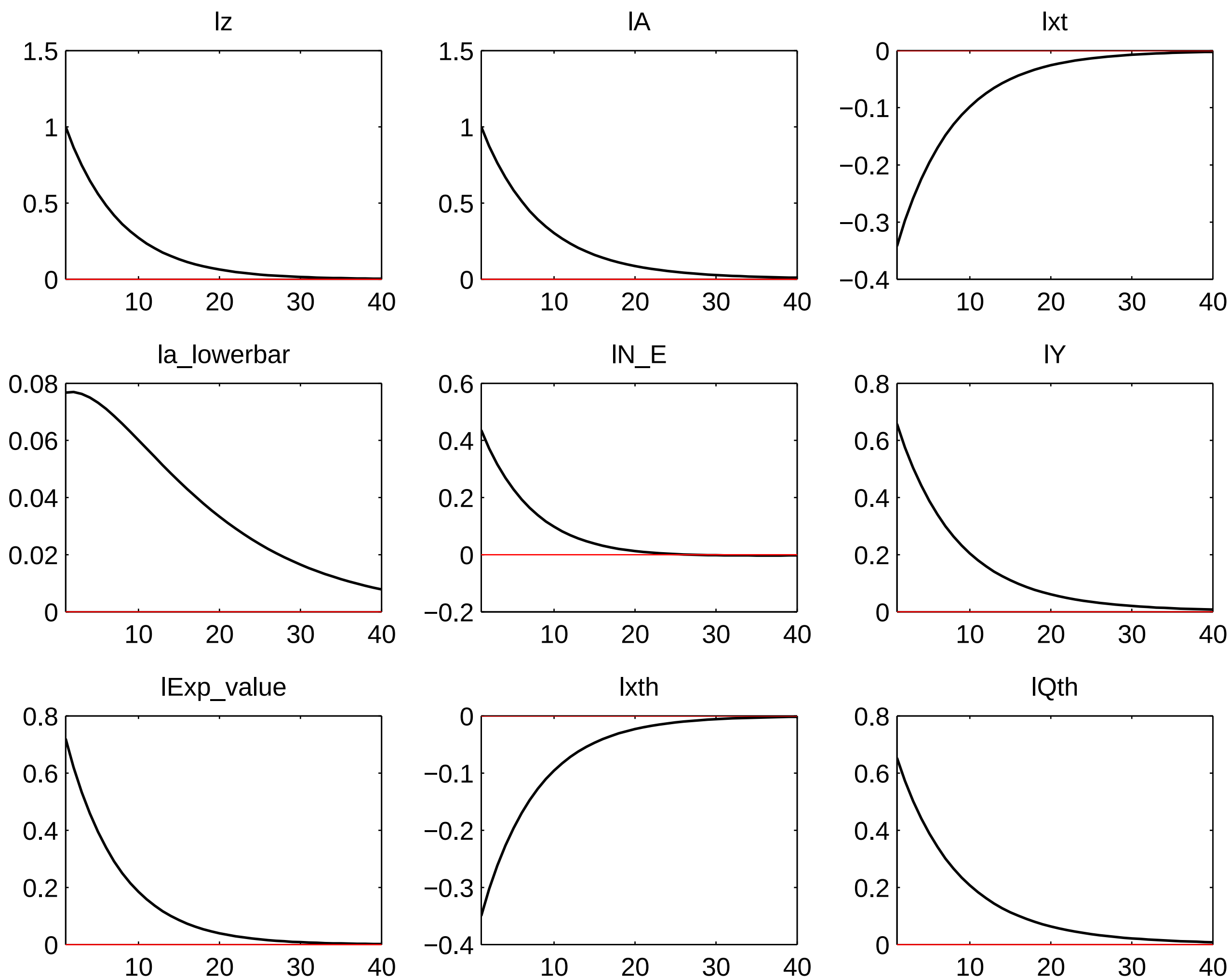
Home bond holding can be found from (22) and (6) such that

$$D_t = \frac{\beta \left(T - G\right)}{1 - \beta},$$

which depends positively on fiscal surplus but independent from N. This implies $RD_t = \left(T - G\right) / \left(1 - \beta\right)$.

Now, to solve for the steady state values, it is enough to solve for the steady state N, which can be found by plugging the steady state cutoff productivity $\underline{a}\left(N\right)$ into the steady state current account balance equation (41) such that

$$\underline{a}\left(N\right) = \left\{ \frac{N}{\epsilon\left(N\right)^{\varphi} Y^{*}} \left[(1 - \gamma_L) w x\left(N\right) - \frac{\xi_t^{*}\left(R^{*} - 1\right)}{\left(1 - \beta R^{*}\right)} c\left(N\right) \right] \right\}^{\frac{\theta - 1}{(\alpha + 1 - \theta)(1 - \varphi)}}. \tag{51}$$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lxt
0
−0.1
−0.2
−0.3
−0.4
10
20
30
40
la_lowerbar
0.08
0.06
0.04
0.02
0
10
20
30
40
lN_E
0.6
0.4
0.2
0
−0.2
10
20
30
40
lY
0.8
0.6
0.4
0.2
0
10
20
30
40
lExp_value
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0.6
0.4
0.2
0
10
20
30
40
lxth
0
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−0.2
−0.3
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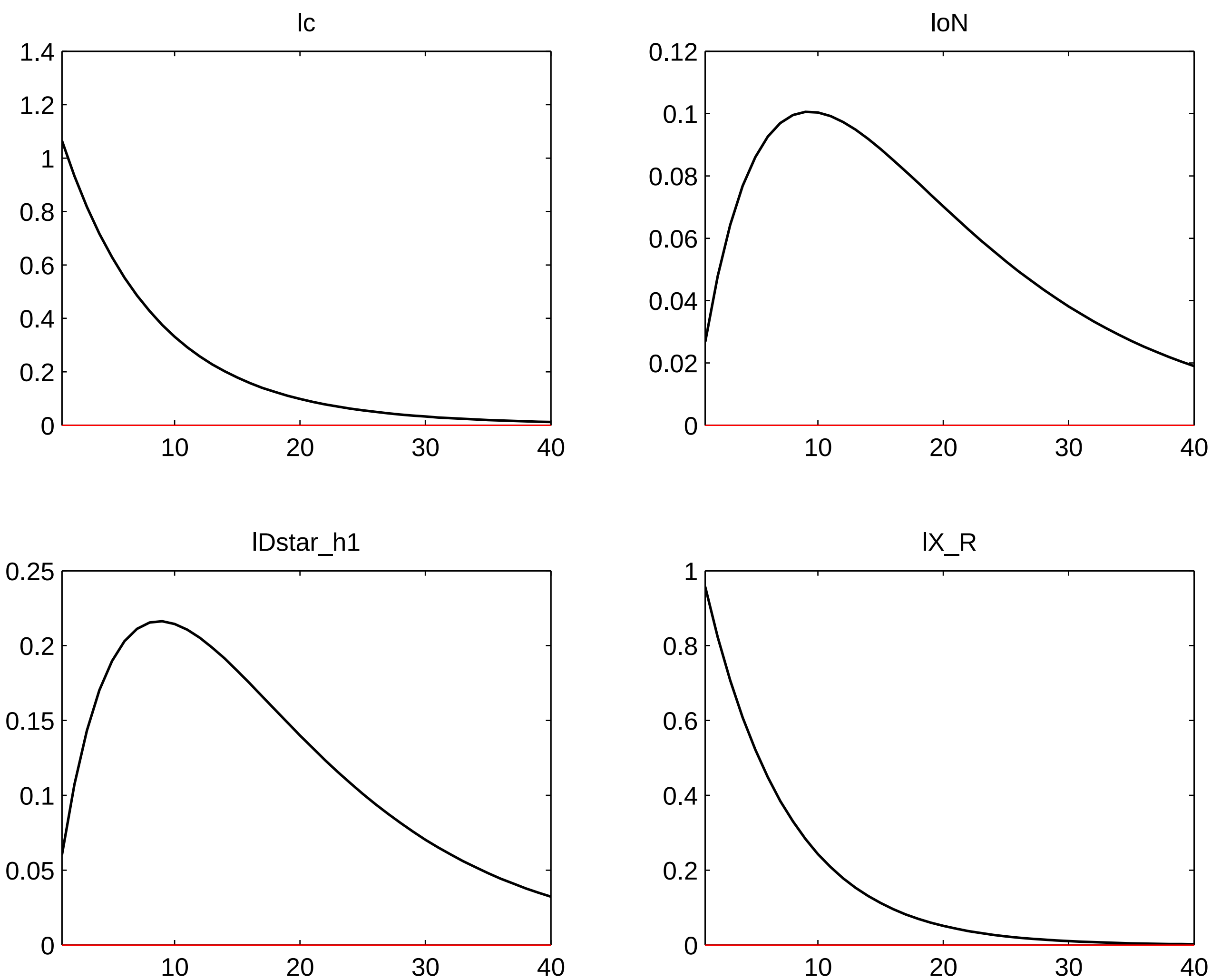
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0
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0.08
0.06
0.04
0.02
0
10
20
30
40
IDstar_h1
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0.2
0.15
0.1
0.05
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0.4
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0
10
20
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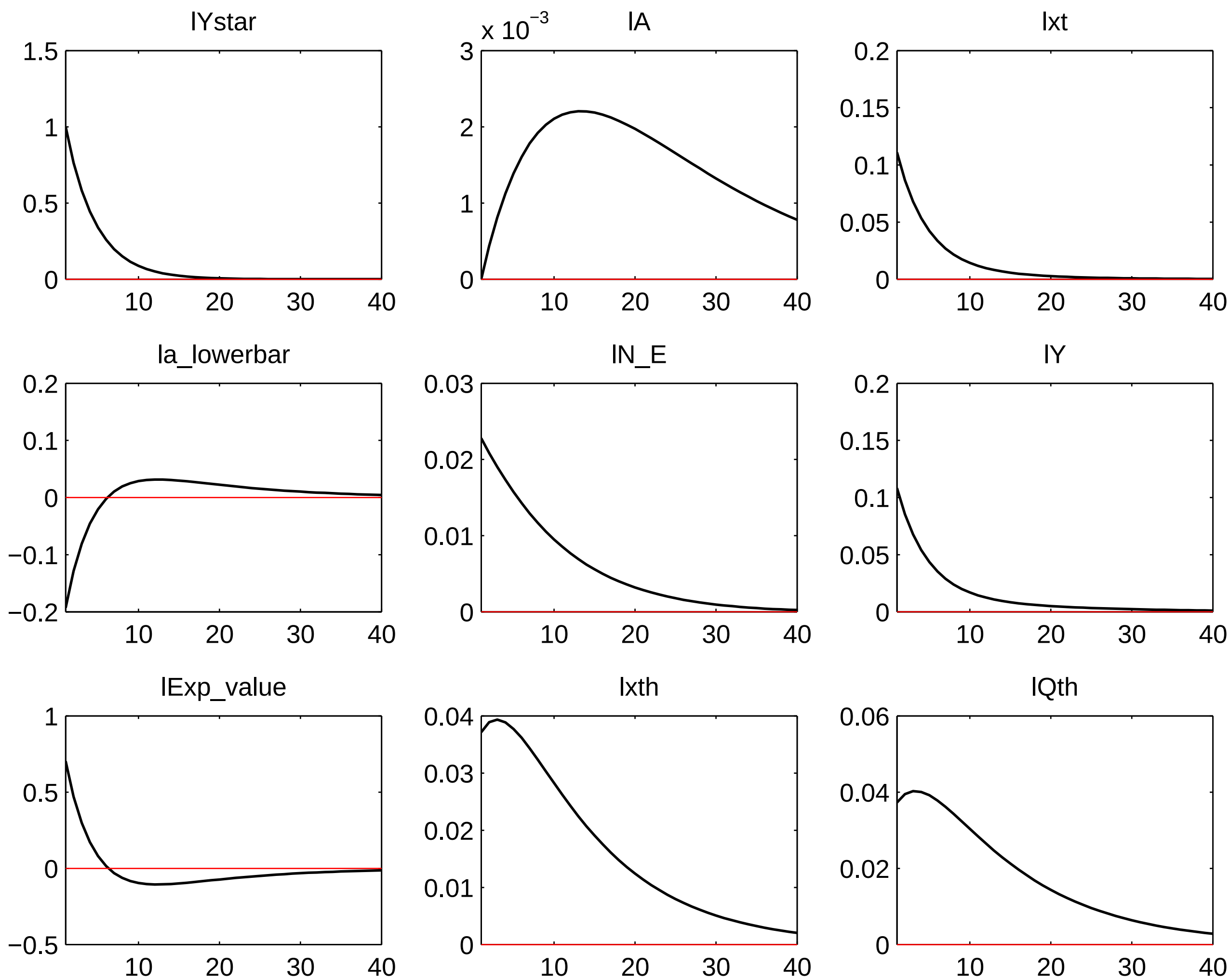
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lxth
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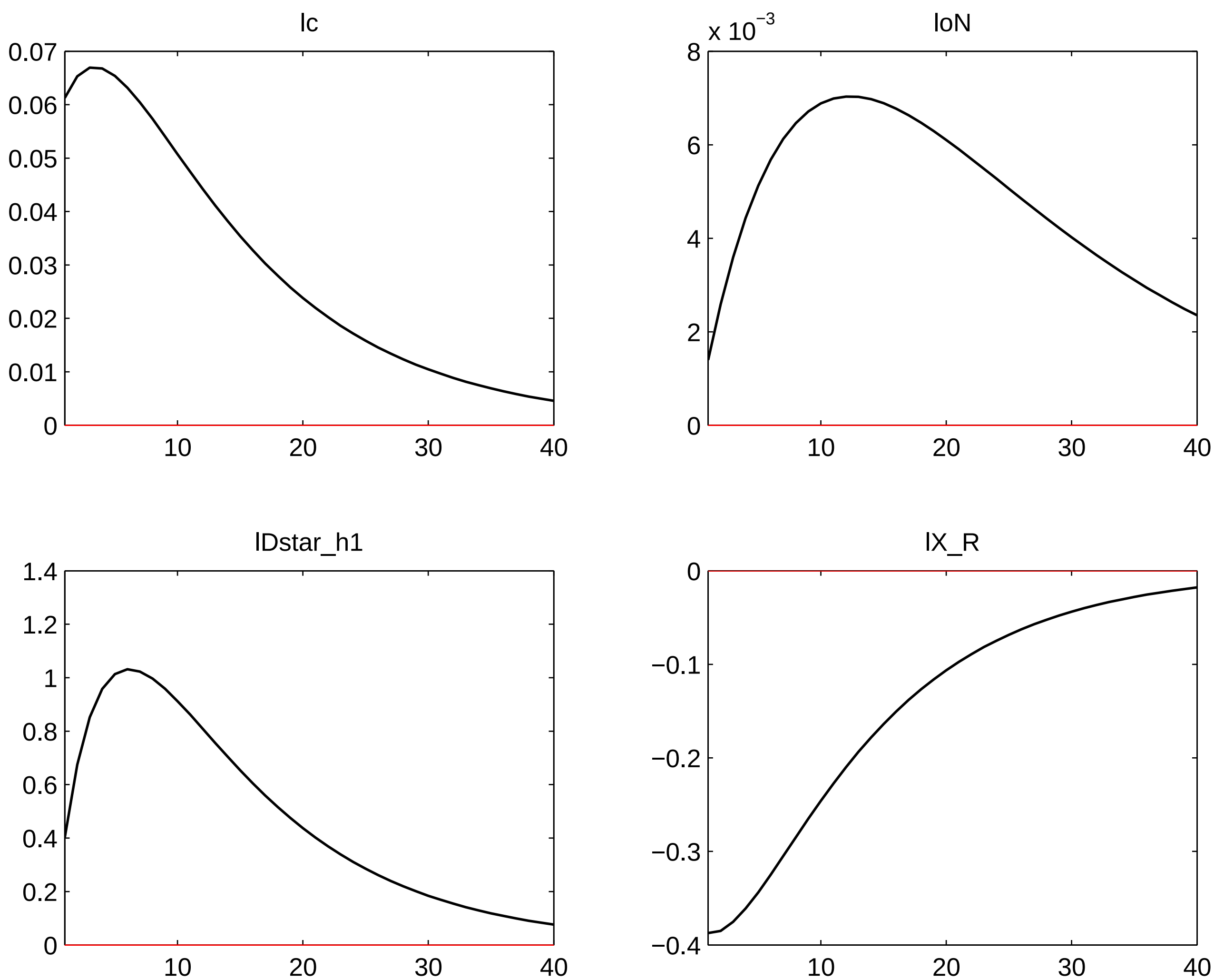
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0.01
0
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20
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40
loN
x 10^−3
8
6
4
2
0
10
20
30
40
lDstar_h1
1.4
1.2
1
0.8
0.6
0.4
0.2
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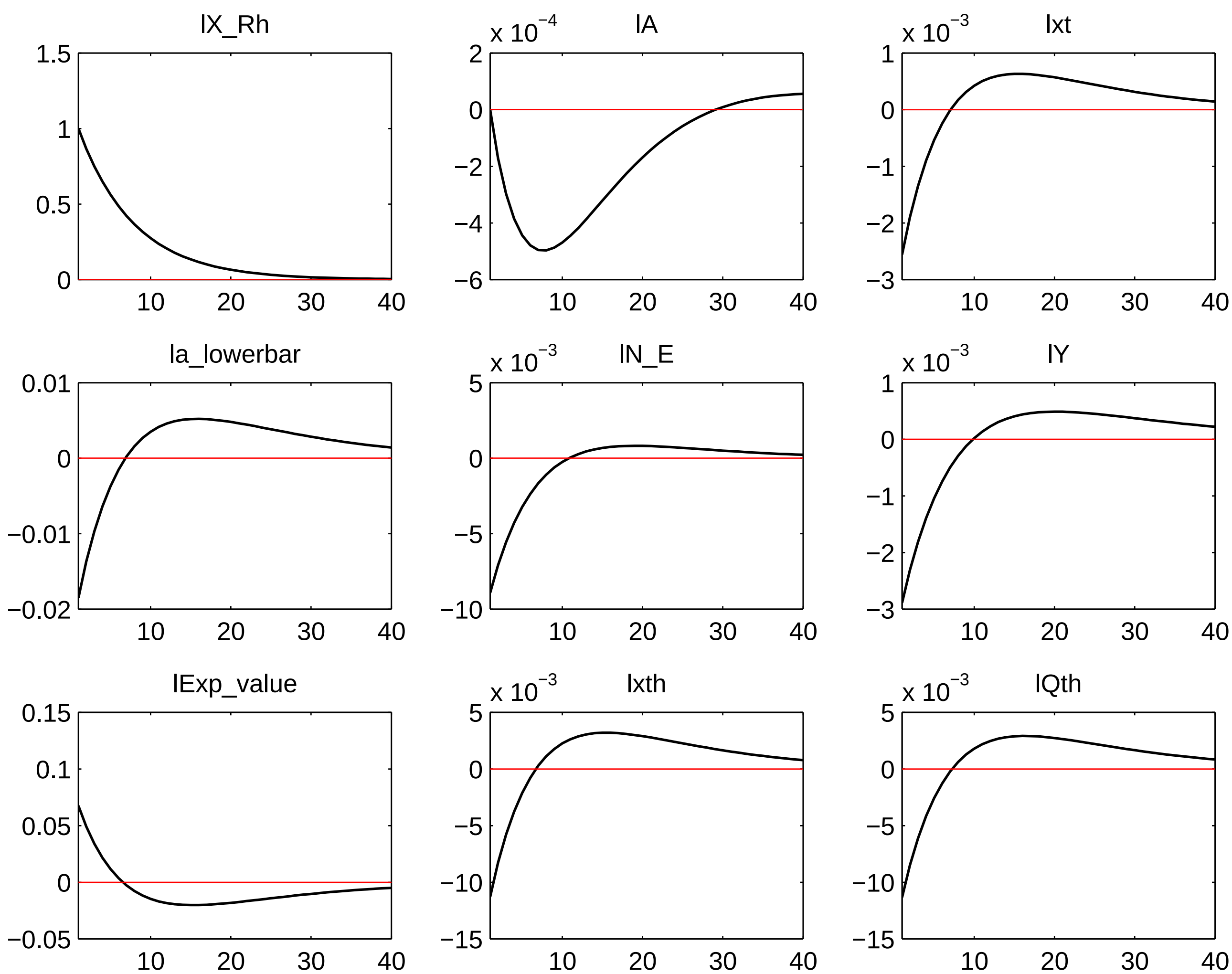
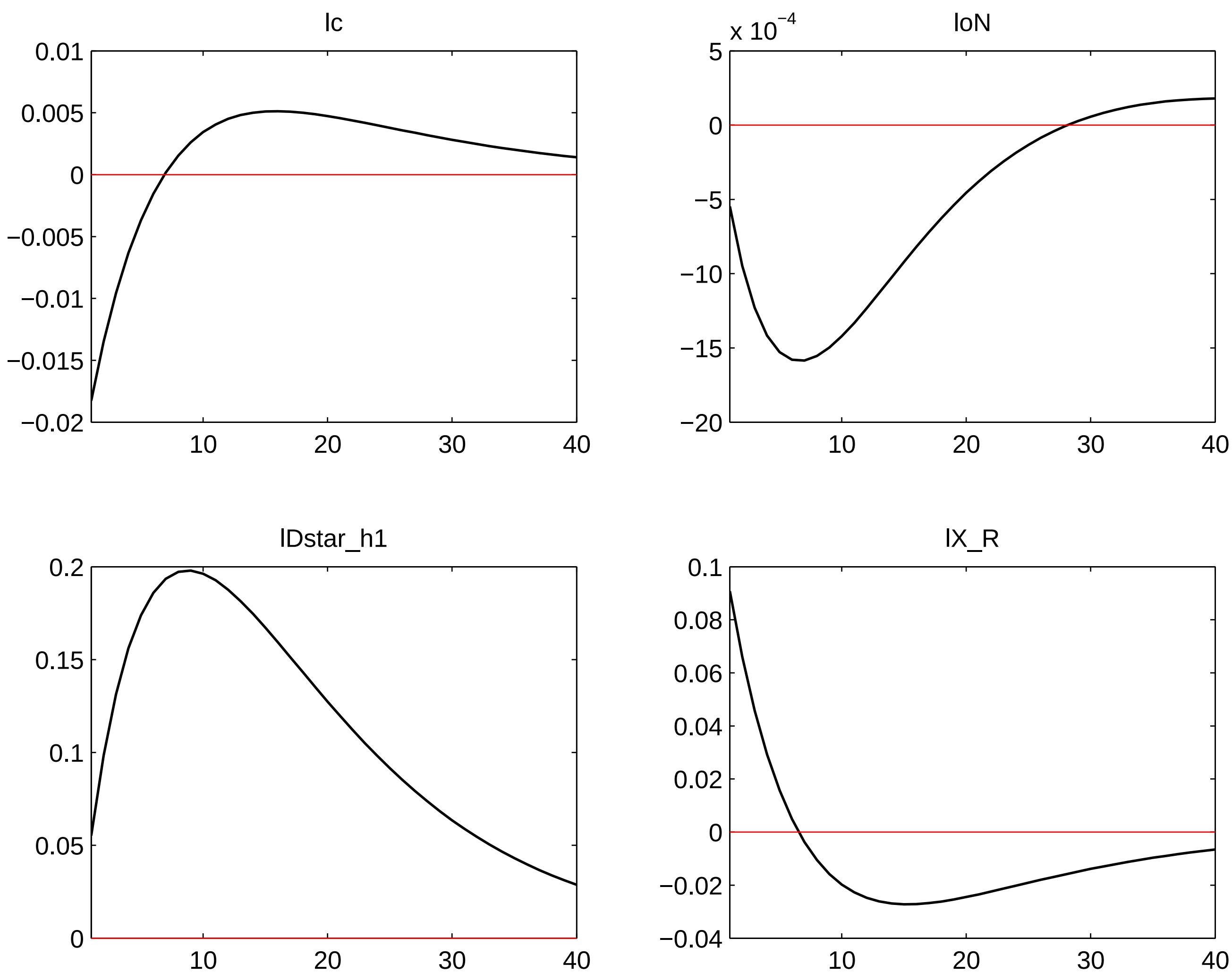


lX_Rh
lA
x 10^−4
lxt
x 10^−3
la_lowerbar
lN_E
x 10^−3
lY
x 10^−3
lExp_value
lxth
x 10^−3
lQth
x 10^−3



lc
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0.005
0
−0.005
−0.01
−0.015
−0.02
10
20
30
40
loN
x 10^−4
5
0
−5
−10
−15
−20
10
20
30
40
lDstar_h1
0.2
0.15
0.1
0.05
0
10
20
30
40
lX_R
0.1
0.08
0.06
0.04
0.02
0
−0.02
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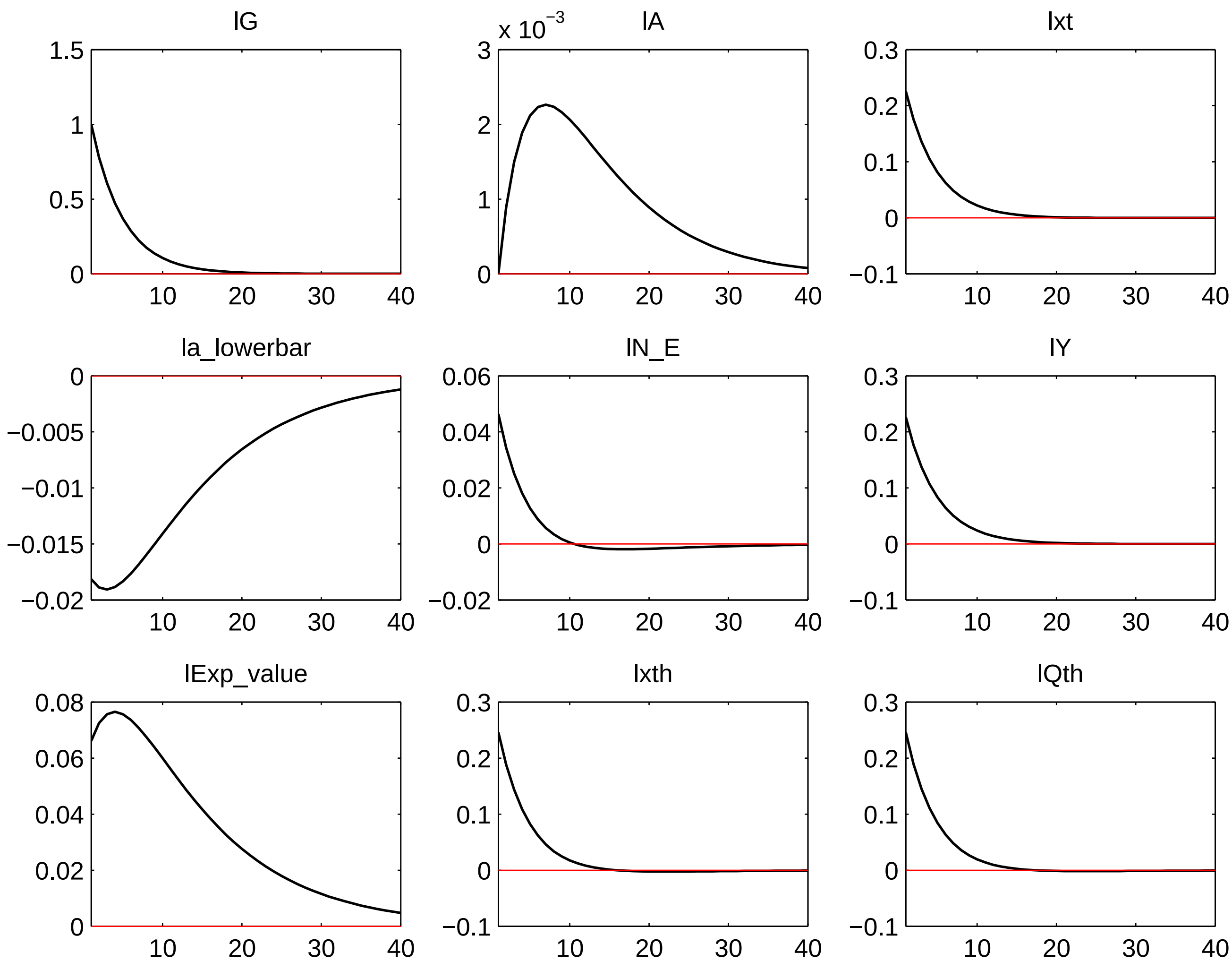
10
20
30
40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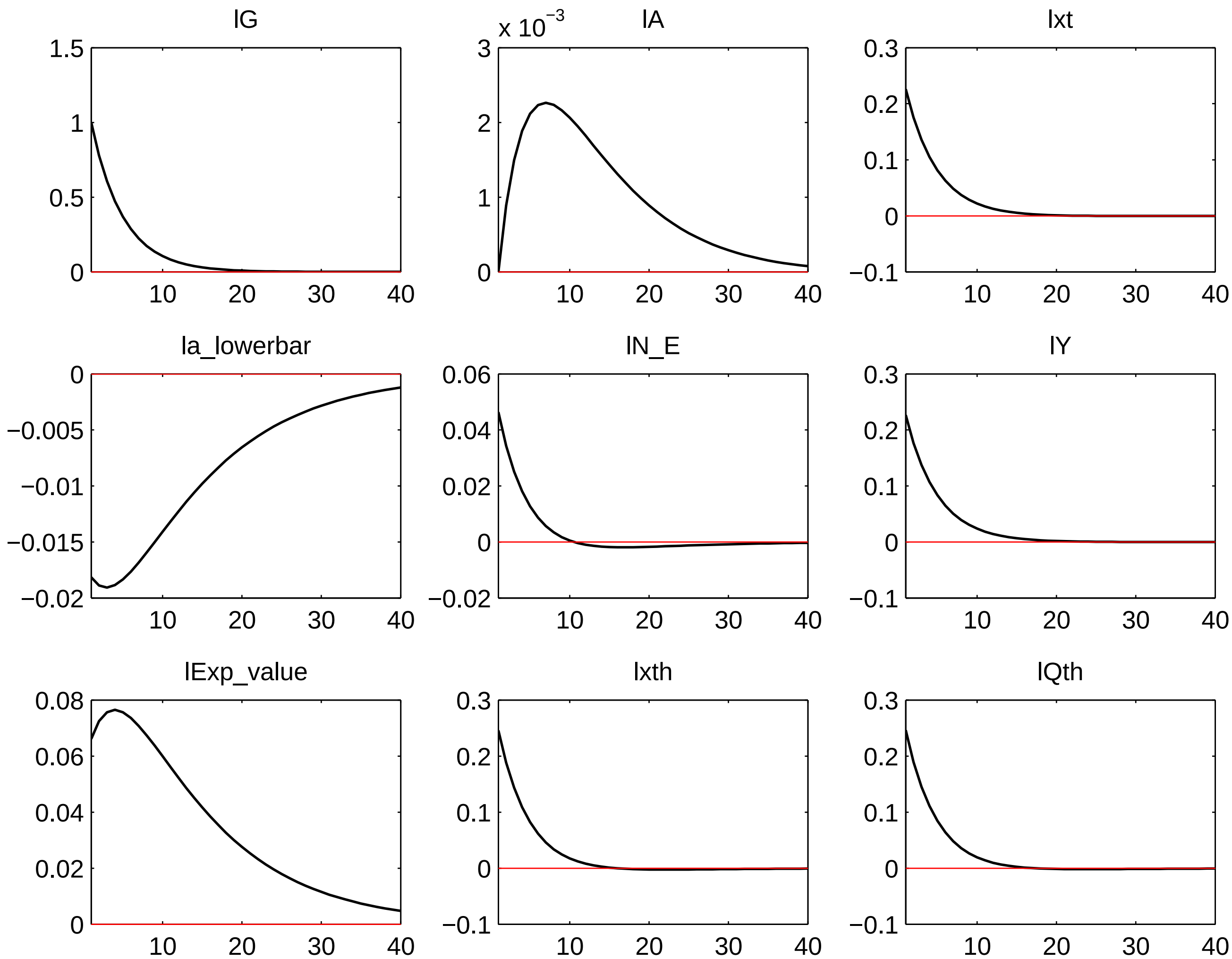
lxt
la_lowerbar
lN_E
lY
lExp_value
lxth
lQth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lQth