Intra-industry Trade and the Gravity Equation: A Cournot-Ricardo Approach

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This paper shows that Cournot competition in segmented markets generate both intra-industry trade and the gravity equation. The paper also demonstrates that the gravity equation holds if and only if the market shares of an exporting country do not depend on the market where it sells. Thus, the widely accepted wisdom that specialization generates the force of gravity is not well grounded on theory.

Key words: Intra-industry trade; Gravity; Cournot
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I. Introduction

Recently, two features of international trade attracted a great deal of attention from economists: gravity and intra-industry trade. Perhaps millions of gravity equations have been estimated since Tinbergen (1962). The regressions all differ in details, but they invariably find that the elasticity of bilateral trade with respect to each partner's income level is roughly equal to 1. At the same time, the high incidence of two-way trade in a single industry has repeatedly been documented since Grubel and Lloyd (1971). Now intra-industry trade is regarded as a norm, at least in trade among industrial countries.

The success of the monopolistic competition model of trade, founded by Dixit and Norman (1980), Krugman (1979) and Lancaster (1979), is largely due to its ability to explain both intra-industry trade and the gravity equation. However, the model has recently been challenged by a number of studies. Davis (1995) shows that by introducing cross-country productivity differences in a Heckscher-Ohlin world, we can easily generate intra-industry trade. More importantly, he makes it clear that specialization in a single industry is responsible for intra-industry trade and what gives rise to specialization does not matter, be it product differentiation or productivity differences or whatever.

On the gravity side also, the exclusive position of the monopolistic competition model has repeatedly been disputed. Hummels and Levinsohn (1995) argue that product differentiation is not likely to be the factor behind the empirical success of the gravity equation since the gravity equation holds well in North-South and South-South trade. They also reemphasize the fact that the gravity equation can be derived from a number of models as long as they generate complete specialization. Deardorff (1998) takes a bolder position that any plausible model would yield the gravity equation. It now seems to be widely accepted - as Grossman (1998) states in his comment on Deardorff (1998) - that specialization –not new or old trade theory - generates the force of gravity in international trade. Specialization seems to be a driving force both behind intra-industry trade and behind the gravity equation.

However, we are familiar with one model in which intra-industry trade prevails with incomplete specialization. Brander (1981) shows that international Cournot competition among producers of an identical commodity can generate intra-industry trade. The model is widely used in a partial equilibrium analysis, but it is rarely applied to trade pattern theory, probably due to the difficulty of incorporating oligopoly in a general equilibrium framework. In the first part of this paper, I merge the Cournotian model of Brander (1981) with the Ricardian model of Dornbusch, Fischer and Samuelson (1977). The purpose of this exercise is to show that this Cournot-Ricardo model of trade also generates the gravity equation. Thus international Cournot competition in segmented markets can generate ***both*** intra-industry trade and the gravity equation. Thus the Cournotian approach to trade should be considered as a serious alternative to monopolistic competition.

The fact that the gravity equation holds in a model with incomplete specialization challenges the common belief that specialization generates the force of gravity. This finding urges us to look for a general condition under which the gravity equation holds, with or without complete specialization. The second part of this paper addresses this question. I demonstrate that the necessary and the sufficient condition for the gravity equation to hold in a world of identical homothetic preferences and frictionless trade is that the market shares of a country be identical across all markets in the world. I show that all known models that generate the gravity equation, including the Cournot-Ricardo model, are just examples of this general condition.

This paper is not the first to show that Cournot competition in homogeneous goods can generate the gravity equation. Song (1998) constructs a general equilibrium two-country model of Cournot competition in which the gravity equation holds.[1] However, the result is obtained under very severe restrictions. This paper extends the model into a multi-country world where the number of firms is endogenously determined.

Feenstra, Markusen and Rose (1998) also demonstrate that the gravity equation can be derived from a model based on Cournot competition. Their approach is more general in that they explicitly consider transportation costs and analyze the home market effects. However, they restrict their analysis to a two-country case. Furthermore, they obtain the gravity equations from regressions on simulated data, not algebraically from the model. In contrast, this paper algebraically derives every result and provides an answer for the question why the gravity equation can emerge in a

[1] This model is identical to the third model of Section II in this paper.

model with incomplete specialization. More importantly, this paper derives a general condition for the gravity equation, which encompasses all known cases of the gravity equation.

The paper is organized as follows. In section II, I construct trade models with a continuum of goods, closely following Dornbusch, Fischer and Samuelson (1977). The analysis of this section parallels Deardorff (1998) in showing that the gravity equation can be easily obtained in a number of models. The simple gravity equation is shown to hold in all these Ricardian models, regardless of whether the underlying market structure is perfect competition, Bertrand competition or Cournot competition. In section III, I derive the necessary and sufficient condition for the gravity equation. The known cases that generate the gravity equation are reinterpreted using this general condition. Section IV concludes the paper.

II. The Pattern of Bilateral Trade in Ricardian Models of Trade

All the models that are considered below are Ricardian in the sense that labor alone is used for production and cross-country differences in technology are respected. We assume a world of frictionless trade with no trade barriers of all sorts. The specifications for technologies and preferences are the same as in the Ricardian model of trade with a continuum of goods by Dornbusch, Fischer and Samuelson (1977-henceforth, the DFS model). However, we vary the market structure from perfect competition to Bertrand competition and to Cournot competition. Our analysis exclusively focuses on the relation between bilateral trade and the income levels of trading partners.

1. Perfect Competition

There exist two countries in the world, and there is a continuum of goods indexed on the unit interval [0, 1]. For each good z on the interval, there are unit labor requirements in the home and in the foreign country, $a(z)$ and $a^*(z)$. Goods are indexed such that home country comparative advantage diminishes as z increases. Defining relative unit labor requirements as

$$A(z) \equiv \frac{a^*(z)}{a(z)},$$

$A(z)$ is decreasing in z. Labor is perfectly mobile between industries. w and w^* denote the competitive wages in the home and in the foreign country, respectively. ω expresses the relative wage of the home country, w/w^*.

The two countries are populated by consumers with identical Cobb-Douglas preferences. Denoting the constant expenditure share on good z by $b(z)$, the demand for good z in each country is determined by the following equations.

$$q(z) = \frac{b(z)\,Y}{p(z)}, \qquad q^*(z) = \frac{b(z)\,Y^*}{p(z)}, \tag{1}$$

where $\int_0^1 b(z)\,dz = 1$.

$q(z)$ and $q^*(z)$ are the demand for good z in the home country and in the foreign country, respectively. Y and Y^* are the income levels of the two countries. $p(z)$ is the price of good z, which, under free trade, is identical in the two countries.

Now we start with the case each industry is perfectly competitive. The model is identical to the DFS model. Let us define the good $\bar{z}(\omega)$ as follows.

$$w = A(\bar{z}). \qquad (2)$$

Then $w\, a(z) \le w^*\, a^*(z)$ for $z \in [0, \bar{z}(w)]$ and $w\, a(z) \ge w^*\, a^*(z)$ for $z \in [\bar{z}(w),1]$. Thus the home country specializes in and exports the range of goods on the interval $[0, \bar{z}(w)]$ and the foreign country specializes in and exports the range of goods on the interval $[\bar{z}(w),1]$. The price of each good is determined by the unit cost in the exporting country.[2]

We can determine the equilibrium relative wage, thus the range of goods produced in each country, by considering labor market equilibrium. In the home country, labor endowment must be equal to the demand for labor, which is the sum of the demand for labor in all industries where the country is producing.

$$L = \int_0^{\bar{z}(w)} a(z)\,(q(z)+q^*(z))\,dz = \int_0^{\bar{z}(w)} a(z)\,\frac{b(z)\,(Y+Y^*)}{w\,a(z)}\,dz = \int_0^{\bar{z}(w)} b(z)\,dz\,\frac{Y+Y^*}{w}. \qquad (3)$$

Similarly, in the foreign country, labor market clearing requires that

$$L^* = \int_{\bar{z}(w)}^{1} a^*(z)\,\frac{b(z)\,(Y+Y^*)}{w^*\,a^*(z)}\,dz = \int_{\bar{z}(w)}^{1} b(z)\,dz\,\frac{Y+Y^*}{w^*}. \qquad (4)$$

Dividing (3) by (4),

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[2] It does not matter how we break the tie on industry $\bar{z}$ as it is a set of measure zero.

$$\frac{L}{L*} = \frac{\int_0^{\bar{z}(w)} b(z)\, dz}{\int_{\bar{z}(w)}^{1} b(z)\, dz} \frac{1}{w}. \tag{5}$$

We can solve equation (5) for w. The left-hand side of equation (5) is the relative supply of labor, while the right-hand side is the relative demand for labor. It is easy to see that the relative demand is monotonically decreasing in w. In addition, it becomes zero at a sufficiently high value of w and becomes infinite at a sufficiently low value of w. Thus a unique solution exists.

The income of the home country is equal to the total expenditure on domestically produced goods. Thus

$$Y = \int_0^{\bar{z}} b(z)\, dz\, (Y+Y*). \tag{6}$$

The value of exports from the home country is equal to the foreign country's demand for the domestically produced goods.

$$X = \int_0^{\bar{z}} b(z)\, dz\, Y*. \tag{7}$$

Combining equations (6) and (7),

$$X = \frac{Y\,Y^*}{Y+Y^*}. \tag{8}$$

Equation (8) is nothing but the simple gravity equation.

2. Bertrand Competition

Now we assume that each industry is a duopoly, composed of a home firm and a foreign firm. Assuming duopoly across industries is certainly a very restrictive condition. The purpose is to derive the most basic model capable of capturing imperfect competition in a general equilibrium framework. We first examine the case where two firms in each industry compete in prices, or engage in Bertrand competition.

As before, for $z \in [0, \bar{z}(w)]$, $w\, a(z)$ £ $w^*\, a^*(z)$. Given the unit-elastic demand in (1), each firm wants to charge the highest possible price as long as it is below the rival's price and gives a non-negative profit. Thus in the Nash equilibrium, the domestic firm who has the lower unit cost charges the unit cost of its rival and captures the entire market. Thus $p(z) = w^*\, a^*(z)$ for $z \in [0, \bar{z}(w)]$[3]. Similarly, $p(z) = w\, a(z)$ for $z \in [\bar{z}(w),1]$. The home country specializes in and exports the range of goods on the interval $[0, \bar{z}(w)]$ and the foreign country specializes in and exports the range of goods on the interval $[\bar{z}(w),1]$. However, the price of each good is not determined by the unit cost in the exporting country, but by the unit cost in the importing country. The difference between the two is the profit margin.

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3 We have an open set problem here. $p(z)$ should be equal to $w^*\, a^*(z)$ - e , where e can be made arbitrarily small.

As before, we can determine w by considering labor markets. In the home country, the labor market is cleared when

$$L = \int_0^{\bar{z}(w)} a(z) \frac{b(z)\,(Y+Y^*)}{w^*\,a^*(z)}\,dz = \int_0^{\bar{z}(w)} \frac{b(z)}{A(z)}\,dz\,\frac{Y+Y^*}{w^*}. \tag{9}$$

Similarly, in the foreign country, labor market clearing requires that

$$L^* = \int_{\bar{z}(w)}^{1} a^*(z) \frac{b(z)\,(Y+Y^*)}{w\,a(z)}\,dz = \int_{\bar{z}(w)}^{1} A(z)\,b(z)\,dz\,\frac{Y+Y^*}{w}. \tag{10}$$

Dividing (9) by (10),

$$\frac{L}{L^*} = \frac{\int_0^{\bar{z}(w)} \frac{b(z)}{A(z)}\,dz}{\int_{\bar{z}(w)}^{1} A(z)\,b(z)\,dz}\,w. \tag{11}$$

Unlike the previous case, the relative demand may not be monotonically decreasing in w. However, it still becomes zero at a sufficiently high value of w and infinite at a sufficiently low value of w. Thus a solution of equation (11) exists, though we have a possibility of multiple equilibria.

Given w, the home country specializes in and exports the range of goods on the interval [0, $\bar{z}$

(w)]. Thus

$$Y = \int_0^{\bar{z}} b(z)\, dz\, (Y+Y^*), \tag{12}$$

and

$$X = \int_0^{\bar{z}} b(z)\, dz\, Y^*. \tag{13}$$

Again we again obtain the simple gravity equation.

$$X = \frac{Y\,Y^*}{Y+Y^*}.$$

3. Cournot Competition

Now we analyze the case where two firms compete in quantities. As in Brander (1981), we assume that firms engage in Cournot competition. The home market and the foreign market for each good *z* are segmented. Each firm makes distinct decisions for each market, treating the other firm's quantity decisions as given. Given the unit-elastic demand curve in (1), it is easy to show that a Cournot-Nash equilibrium in the home market requires the following conditions to hold.

$$p(z)\,(1\text{-}s(z)) = w\,a(z),$$

$$p(z)\,(1\text{-}s^*(z)) = w^*\,a^*(z). \tag{14}$$

The left-hand side of each equation is the marginal revenue of each firm, and the right-hand side is the unit cost of each firm. s is the market share of the home firm and s^* is the market share of the foreign firm. Noting that the sum of s and s^* should be equal to 1, the above equations can be solved to generate the following equilibrium conditions.

$$p(z) = w\, a(z) + w^*\, a^*(z),$$

$$s(z) = \frac{w^*\, a^*(z)}{w\, a(z) + w^*\, a^*(z)} = \frac{A(z)}{A(z) + \omega}\ . \tag{15}$$

p is determined by the sum of the two firms' unit costs. The market is divided between the two firms according to s, which is a measure of the home firm's ***competitive*** advantage. This measure is increasing in the home country's relative productivity $A(z)$ and is decreasing in its relative wage ω.

Repeating the same procedure for the foreign market, one can easily check that the price there is also equal to the sum of the two firms' unit costs, and the market share of each firm is identical in the home and the foreign market. Thus, the world supply of the home firm in industry z is given by

$$s(z)\ (q(z) + q^*(z)) = s(z)\, \frac{b(z)\ (Y + Y^*)}{p(z)}, \tag{16}$$

where $p(z)$ and $s(z)$ are given by the equations in (15). Likewise, the world supply of the foreign firm in industry z is equal to

$$s^*(z)\ (q(z) + q^*(z)) = (1 - s(z))\, \frac{b(z)\ (Y + Y^*)}{p(z)}. \tag{17}$$

Let us define $E[s]$ as the mean of $s(z)$ over the commodity interval, where the density is given by the expenditure share $b(z)$.

$$E[s] \equiv \int_0^1 s(z)\, b(z)\, dz. \tag{18}$$

The labor market clearing condition for the home country is given by

$$L = \int_0^1 a(z)\, s(z)\, (q(z) + q^*(z))\, dz$$

$$= \int_0^1 s(z) \frac{w\, a(z)}{w\, a(z) + w^*\, a^*(z)}\, b(z)\, dz\, \frac{Y+Y^*}{w}$$

$$= \int_0^1 s(z)\, (1-s(z))\, b(z)\, dz\, \frac{Y+Y^*}{w}. \tag{19}$$

Similarly, in the foreign country,

$$L^* = \int_0^1 a^*(z)\, (1-s(z))\, (q(z) + q^*(z))\, dz$$

$$= \int_0^1 (1-s(z)) \frac{w^*\, a^*(z)}{w\, a(z) + w^*\, a^*(z)}\, b(z)\, dz\, \frac{Y+Y^*}{w^*}$$

$$= \int_0^1 (1-s(z))s(z)\, b(z)\, dz\, \frac{Y+Y^*}{w^*}. \tag{20}$$

Dividing (20) by (19),

$$\boldsymbol{w} = \frac{w}{w^*} = \frac{L^*}{L}. \tag{21}$$

Equation (22) sets the relative wage as a function of the relative labor endowment alone.

Utilizing equation (21), the market share of the home country in equation (15) can be expressed as a function of the two parameters: comparative advantage and relative labor endowment.

$$s(z) = \frac{A(z)}{A(z) + L^*/L}\,. \tag{22}$$

The national income of the home country is given by the total expenditure on domestically produced goods.

$$Y = \int_0^1 s(z)\, b(z)\, dz\, (Y+Y^*) = E[s]\, (Y+Y^*). \tag{23}$$

Thus, $E[s]$, which is the mean of the home country market shares, also equals the share of home GDP in world GDP.

A notable feature of the equilibrium is that a country produces every type of good, regardless of its comparative advantage and its relative wage. In other words, no specialization exists. Comparative advantages affect only the distribution of a country' s market shares across industries.

Intra-industry trade is also generated in each industry as in Brander (1981). In each industry, the size of the home market is given by $b(z)\, Y$, and that of the foreign market is given by $b(z)\, Y^*$. The home firm supplies the fraction $s(z)$, while the foreign firm supplies the fraction $1\text{-}s(z)$. Thus, the value of exports from the home country is

$$x(z) = s(z)\, b(z)\, Y^* = s(z)\, b(z)\, (1\text{-}E[s])\, (Y+Y^*). \tag{24}$$

The value of imports of the home country is

$$m(z) = (1\text{-}s(z))\, b(z)\, Y = (1\text{-}s(z))\, b(z)\, E[s]\, (Y{+}Y^*). \tag{25}$$

Now the total value of exports from the home country can be obtained from equation (24).

$$X = \int_0^1 x(z)\, dz = \int_0^1 s(z)\, b(z)\, (1\text{-}E[s])\, (Y{+}Y^*)\, dz = E[s]\, (1\text{-}E[s])\, (Y{+}Y^*). \tag{26}$$

Plugging equation (23) into equation (26),

$$X = \frac{Y\,Y^*}{Y{+}Y^*}.$$

We again obtain the simple gravity equation.

4. General Cournot Competition

Now let us generalize the previous model. Now assume that there are N countries in the world indexed by i. Every consumer in the world has an identical CES utility function with the constant elasticity given by e. In industry z, country i has $f_i(z)$ firms, whose unit costs are all equal to $w_i\, a_i(z)$, where w_i is the wage rate in country i and $a_i(z)$ is the unit labor requirement in country i. The N markets in the world are mutually segmented and in each market, $\sum_{i=1}^{N} f_i(z)$ firms play a distinct Cournot game. The Cournot-Nash equilibrium in country j' s market requires that

$$p_j(z)\,(1 - \frac{s_{ij}(z)}{e}) = w_i\, a_i(z) \quad \text{for every firm in country } i \text{ and for every } i. \tag{27}$$

The left-hand side is the marginal revenue of a country i firm and the right-hand side is the marginal cost of a country i firm. $\boldsymbol{s}_{ij}(z)$ denotes the market share of a country i firm in country j' s market for good z. Let us denote the collective market share of country i firms by $s_{ij}(z)$. Then

$$s_{ij}(z) = f_i(z)\ \boldsymbol{s}_{ij}(z), \tag{28}$$

$$\sum_{i=1}^{N} s_{ij}(z) = 1. \tag{29}$$

Multiplying both sides of equation (27) by $f_i(z)$ and summing over i using equations (28) and (29),

$$p_j(z) = \frac{\sum_{i=1}^{N} f_i(z)\ w_i\, a_i(z)}{\sum_{i=1}^{N} f_i(z) - \frac{1}{\boldsymbol{e}}}, \tag{30}$$

$$s_{ij}(z) = (1 - \frac{w_i\, a_i(z)}{p_j(z)})\ f_i(z)\ \boldsymbol{e}.^{4} \tag{31}$$

If the number of firms in each country is exogenous, the labor market clearing conditions in the N countries will determine w_i. If it is endogenously determined, we will have to introduce fixed costs explicitly. Then the labor market clearing conditions and the zero-profit conditions will determine w_i and $f_i(z)$. This would be a very complicated process, and unlike in the previous case, we cannot explicitly show how a solution can be obtained. Thus we will just assume that a solution exists.

Whatever the solution is, we can see from equation (30) that $p_j(z)$ is identical across the world

[4] These equations assume that every country produces a positive amount. However, for some countries the market shares can be zero. Then the summation in equation (30) should be over the countries that produce a

without any arbitrage. The same set of firms compete in every market with the unit costs that do not vary with the location of the market. Furthermore, the demand elasticity, which is crucial in determining the mark-up over the marginal cost, is identical across the world. Thus the equilibrium price is identical across the N markets. In equation (31), we can also see that $s_{ij}(z)$ does not depend on j. Let us denote this common share by $s_i(z)$. The market share of each country is identical across the N markets. These properties hold in every industry.

Let $b(z)$ be the expenditure share on good z in equilibrium.[5] Then the sales of country i firms in the country j' s market for good z will be given by

$$X_{ij}(z) = s_i(z)\ b(z)\ Y_j. \tag{32}$$

Y_j is the income level of country j. Integrating both sides of equation (32) over z yields the following equation.

$$X_{ij} = E[s_i]\ Y_j. \tag{33}$$

X_{ij} is the exports of country i to country j.[6] $E[s_i]$ is defined as follows.

$$E[s_i] \equiv \int_0^1 s_i(z)\ b(z)\ dz. \tag{34}$$

Now make a summation over j of both sides of equation (33).

strictly positive amount.

5 With a CES utility function, this would depend on equilibrium prices. As these are identical across countries, $b(z)$ also is identical across countries.

6 If j equals i, it equals the total shipment of country i to its own markets.

$$Y_i = \sum_{j=1}^{N} X_{ij} = E[s_i]\ Y_W. \qquad (35)$$

The income of country i must be equal to its total sales to the world (including itself). Y_W is the world income. Thus $E[s_i]$ is the share of country i in the world income. Plugging this into equation (33), we finally get

$$X_{ij} = \frac{1}{Y_W} Y_i\ Y_j. \qquad (36)$$

We obtain the simple gravity equation for the N country world.

III. Sufficient Conditions for the Gravity Equation[7]

It is now well understood that in a world of identical homothetic preferences and zero trade barriers, complete specialization (each good is produced in only one country) generates the gravity equation. Complete specialization can occur in a number of trade models. Be it Armington assumption as in Anderson (1979) or increasing returns to scale in a world of product differentiation or large factor endowment differences in a Heckscher-Ohlin world, complete specialization generates gravity. As Grossman (1998) comments, "specialization – and not new trade theory or old trade theory - generates the force of gravity."

Thus it is not surprising at all that the gravity equation holds under the first two models considered in the previous section. In the case of perfect competition and Bertrand competition, complete specialization occurs in equilibrium. The gravity equation must hold in the two cases.

However, our Cournot-Ricardo models (model 3 and model 4) defy the conventional wisdom

7 A part of material in the section overlaps with Song (2000).

that specialization generates the force of gravity. In these models, firms from multiple countries are producing in a single industry. There is no complete specialization, but the force of gravity still commands. Why?

To answer this question, we have to identify a general condition under which the gravity equation holds, with or without complete specialization. For this purpose, suppose again that the world is populated by consumers with identical homothetic preferences defined on the commodity interval of [0,1]. There is no friction in trade and goods prices are equalized across the world. We use the same notations as before. Let us define the following variables.

$$E[s_{ij}] \equiv \int_0^1 s_{ij}(z)\, b(z)\, dz, \tag{37}$$

$$E[s_i] \equiv \sum_{j=1}^{N} E[s_{ij}]\, \frac{Y_j}{Y_W}. \tag{38}$$

$E[s_{ij}]$ is the mean of $s_{ij}(z)$ over the commodity interval, the density being given by the expenditure share. $E[s_i]$ is the mean of $E[s_{ij}]$ over j, where the density is given by the share of an importing country in the world income. It is easy to see that

$$X_{ij}(z) = s_{ij}(z)\, b(z)\, Y_j, \tag{39}$$

$$X_{ij} = \int_0^1 X_{ij}(z)\, dz = E[s_{ij}]\, Y_j, \tag{40}$$

$$Y_i = \sum_{j=1}^{N} X_{ij} = \sum_{j=1}^{N} E[s_{ij}] \frac{Y_j}{Y_W} Y_W = E[s_i]\ Y_W. \qquad (41)$$

Combining equations (40) and (41), we obtain equation (42).

$$X_{ij} = \frac{E[s_{ij}]}{E[s_i]} \frac{Y_i\ Y_j}{Y_W}. \qquad (42)$$

Now we can prove the following proposition.

Proposition 1]

$X_{ij} = \frac{1}{Y_W} Y_i\ Y_j$ for every i and j if $s_{ij}(z)$ is independent of j for every i and z.

The proof is very simple. If the condition holds, $E[s_{ij}]$ is independent of j in equation (37). Then it is easy to see in equation (38) that $E[s_i] = E[s_{ij}]$ for every i and j. By equation (42), the simple gravity equation holds for every i and j.

In other words, if the market shares of each exporting country in importing countries are identical across the world, the gravity equation holds. Actually, this condition is also necessary in the following sense. Suppose there exist some i, j and k such that $s_{ij}(z) \neq s_{ik}(z)$ on a non-negligible set of z. Then we can find some preferences under which $E[s_{ij}] \neq E[s_{ik}]$. Then at least one of them differs from $E[s_i]$ and according to equation (42), the gravity equation fails to hold for that country pair. Thus for the gravity equation to hold under arbitrary preferences, the market shares of each exporting country should be identical across the world in almost all industries.

Therefore, the equality of the market shares – not specialization – generates the force of gravity. We can easily see that all cases where the gravity equation holds is a special example of the general condition that we identified.

Example 1] *Complete Specialization*

Let $Z(i)$ be the set of goods that a country i produces. Then $Z(i)$ would be mutually exclusive. Thus, for $z \in Z(i)$, $s_{ij}(z) = 1$ for every j, and for $z \notin Z(i)$, $s_{ij}(z) = 0$ for every j. The gravity equation holds because the market shares of a country are identically equal to 1 or zero across the world.

Example 2] *Random Selection*

This is an example considered by Deardorff (1998), which can be traced back to Leamer and Stern (1970). According to this model, shoppers in the world come to a common world pool of good z. Since all goods are equally priced, a shopper is indifferent among them and randomly selects producers. Let $Y_i(z)$ be the production of good z in country i and $Y_W(z)$ be the production of good z in the world. Then the probability that a shopper from country j selects a country i producer is equal to $Y_i(z)/Y_W(z)$. Thus the share of country i firms in the basket of shoppers from country j would be equal to this number. Since it does not depend on j, our condition holds and the gravity equation holds.

Example 3] *Cournot Competition*

Now we go back to the Cournot-Ricardo model in the previous section. Recall that $s_{ij}(z)$ does not depend on j . Now we understand why the gravity equation holds without complete specialization.

Example 4] *Hybrid Case*

What matters for the gravity equation is that the market shares of an exporting country should not vary with importing countries. The common market share does not have to be 1 or 0. As the examples above show, it can be any number between 0 and 1. Furthermore, the common market share can vary across industries. Thus the gravity equation holds in a model where complete specialization occurs in a subset of industries, and random selection occurs in another subset and Cournot competition prevails elsewhere.

The random selection model of Deardorff (1998) is considered by many as the only model with incomplete specialization that yields the gravity equation. This model anticipates the result of this paper by showing that specialization is not necessary for gravity. Furthermore, intra-industry trade emerges in equilibrium. However, many observers, including Grossman (1998), seriously question the relevance of this model in reality. The reason seems to be compelling. A tiny amount of trade friction would totally reshuffle the equilibrium trade flows.

The Cournot-Ricardo model in Section II provides another model with incomplete specialization that yields the gravity equation. However, this model is immune from the kind of criticism mentioned above. If we introduce transportation costs into the model, the prices and the market shares would vary with the markets. However, if we let these costs go to zero, the model will converge to the equilibrium in section II. A small amount of trade friction would not fundamentally alter the force of gravity. This is not the case with the random selection model.

IV. Conclusion

Probably Deardorff (1998) was not quite right when he said that any plausible model would yield the gravity equation. However the gravity equation does hold under a very general condition. It holds in a traditional Ricardian world and also in a world of Bertrand competition. The gravity equation also holds in a world of Cournot competition with incomplete specialization.

It implies that complete specialization is not necessary for the gravity equation. This paper finds that what is really necessary is the condition that the market shares of a country do not depend on in which market it sells. The Cournot-Ricardo model satisfies this condition, as all other known cases that yield the gravity equation do.

International Cournot competition in segmented markets can generate ***both*** intra-industry trade and the gravity equation. While product differentiation seems to be a way of life in most industries, it is often unclear whether it is product differentiation or some strategic force that is driving intra-industry trade. Bernhofen (1998) finds a good fit between the Brander (1981) model and data in petrochemical products. Choi (1999) finds that intra-industry trade prevails in hot-rolled steel products, down to the 6-digit industry classification. These products are highly standardized and almost identical. The Cournotian approach to trade should be considered as a serious alternative to monopolistic competition.

The gravity equations are quite frequently employed in the analysis of international economic policies. For example, if one finds that bilateral trade flows among some countries are greater than a fitted gravity equation predicts, these countries are often interpreted to be natural trading partners to each other and a preferential trade arrangement among them is often suggested. Theoretical economists have always been skeptical about this kind of procedure since the gravity equation lacks theoretical justification and the kinds of products exchanged may be more responsible here than the characteristics of the countries involved. This paper suggests that one may be more aggressive in the use of the gravity equations for policy recommendation as they tend to hold under a quite general condition and across most kinds of products.

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