Diagnostic Expectations and Emerging Market Business Cycles

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Introduction

- Goal: Study linear dynamic SOE models with “Diagnostic Expectations” (DE)
- What are diagnostic expectations?
  - Based on “representativeness heuristic” (Kahneman & Tversky)
  - Tendency to exaggerate how representative a small sample is
  - Informed by human memory studies, addressing memory flaws (Gennaioli & Shleifer 2010; Kahana 2012)
- Dynamic settings: Impact overreaction & extrapolation

Why Diagnostic Expectations?

1. A well-established psychological foundation
   - Both in psychology and economics

2. Consistent with individual survey forecasts
   (Bordalo, Gennaioli, Ma & Shleifer, AER 2020)

3. A portable, tractable model of beliefs
   - In closed economy linear DSGE models
     (Bianchi, Ilut & Saijo 2023; L'Huillier, Singh & Yoo 2023)

4. A micro-founded model of beliefs
   - Survives Lucas critique

DE can deliver useful insights in dynamics open economy models

A) Theoretically, address 3 issues

1. Countercyclical impact trade balance
2. Endogenous, repeated booms and busts in capital flows
3. ~~Investment channel for stronger countercyclical trade balance~~

B) Empirically

- DE increase the impact of temporary TFP shocks
- DE generate endogenous demand shifter

Outline

1. Introduction
2. **Preliminaries: Overview of Diagnostic Expectations**
3. Theoretical: Endowment economy
4. Empirical: Quantitative SOE-DSGE models

Related Literature

- Diagnostic expectations

  Kahneman & Tversky (1972); Kahneman, Slovik & Tversky (1982); Bordalo, Gennaioli & Shleifer (BGS) (JF 2018); Bordalo, Gennaioli, La Porta & Shleifer (JF 2019); Bordalo, Gennaioli, Ma & Shleifer (AER 2020); Bordalo, Gennaioli, Shleifer & Terry (2020); D'Arienzo (2020); Bordalo, Kauffman, Gennaioli & Shleifer (AER 2019); Maxted (2021); Bianchi, Ilut & Saijo (2023); L'Huillier, Singh & Yoo (2023),...

- Behavioral macroeconomics

  Angeletos & Lian (AER 2018); Gabaix (AER 2020); Farhi and Werning (AER 2019); Garcia-Schmidt & Woodford (AER 2019); Lian (2020); Angeletos, Huo & Sastry (2020); Woodford (AEA P&P 2019); Gust, Herbst & López-Salido (2020); Bianchi-Vimercati, Eichenbaum & Guerreiro (2022);...

- Small open economy business cycles

  Medoza (1991); Aguiar and Gopinath (2007); Garcia-Cicco, Pancrazi & Uribe (2010);...

Representativeness Heuristic

- The distribution of Irish hair color (Bordalo, Gennaioli & Shleifer 2020)

	T=red	T=blond/light brown	T=dark brown
G≡ Irish	10%	40%	50%
G≡ World	1%	14%	85%

- Idea: Agents tend to react more to representative attribute
  - "An attribute is representative of a class if the relative frequency of this attribute is much higher in that class than in the relevant reference class" (Kahneman & Tversky)

- The most representative Irish hair color is red as

$$\frac{h(T=t|G)}{h(T=t|-G)} = \frac{Pr(\text{red hair}|\text{Irish})}{Pr(\text{red hair}|\text{World})} = \frac{10\%}{1\%} = 10$$

Diagnostic Expectations

- Consider the process

$$x_t = \rho_x x_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma_\varepsilon^2)$$

- Diagnostic pdf is defined as

$$f_t^\theta(x_{t+1}) = \underbrace{f(x_{t+1}|G_t)}_{\text{true pdf}} \cdot \underbrace{\left[\frac{f(x_{t+1}|G_t)}{f(x_{t+1}|-G_t)}\right]^\theta}_{\text{distortion}} \cdot C, \quad \theta > 0$$

- Information sets:
  - G_t: current state t
  - $-G_t$: reference state, here $t-1$.

θ: degree of diagnosticity

Formula for Univariate Case and Example

- Diagnostic expectation is:

$$\mathbb{E}_t^\theta[x_{t+1}] = \mathbb{E}_t[x_{t+1}] + \theta(\mathbb{E}_t[x_{t+1}] - \mathbb{E}_{t-1}[x_{t+1}])$$

(Bordalo, Gennaioli & Shleifer 2018, henceforth BGS)

- We have that:

$\mathbb{E}_t[x_{t+1}] = \rho_x x_t$ and $\mathbb{E}_{t-1}[x_{t+1}] = \rho_x^2 x_{t-1}$

- So:

$$\mathbb{E}_t^\theta[x_{t+1}] = \rho_x x_t + \theta(\rho_x x_t - \rho_x^2 x_{t-1}) = \rho_x x_t + \theta\rho_x\varepsilon_t$$

$\Longrightarrow$ excess volatility in beliefs on impact

DE with distant memory

- DE when reference period is in *distant past*

$$\mathbb{E}_t^{\theta,J}[x_{t+1}] = \mathbb{E}_t[x_{t+1}] + \theta \underbrace{\left(\mathbb{E}_t[x_{t+1}] - \sum_{j=1}^{J} \alpha_{j,J}\mathbb{E}_{t-j}[x_{t+1}]\right)}_{\text{weighted average of forecast revisions}}; \quad \sum_{j=1}^{J} \alpha_{j,J} = 1$$

- For $J = 1, 2, ...$

$$\mathbb{E}_t^{\theta,1}[x_{t+1}] = \mathbb{E}_t[x_{t+1}] + \theta\left(\mathbb{E}_t[x_{t+1}] - \mathbb{E}_{t-1}[x_{t+1}]\right)$$

$$\mathbb{E}_t^{\theta,2}[x_{t+1}] = \mathbb{E}_t[x_{t+1}] + \theta\left(\mathbb{E}_t[x_{t+1}] - \sum_{j=1}^{2} \alpha_{j,2}\mathbb{E}_{t-j}[x_{t+1}]\right)$$

$\theta = 0$ corresponds to Rational Expectations (RE)

Diagnostic PDF with distant memory

DE formula with distant memory

- With equal weights to the past:

$$\mathbb{E}_t^{\theta,J}[x_{t+1}] = \mathbb{E}_t[x_{t+1}] + \theta\left(\mathbb{E}_t[x_{t+1}] - \frac{1}{J}\sum_{j=1}^{J}\mathbb{E}_{t-j}[x_{t+1}]\right)$$

$$= \underbrace{\mathbb{E}_t[x_{t+1}]}_{\text{RE}} + \theta\rho\sum_{j=0}^{J-1}\left(\frac{J-j}{J}\right)\rho^j\varepsilon_{t-j}$$

- Agents extrapolate past shocks into the future
- Impact of past shocks decaying over time

Endogenous extrapolation

- Diagnostic expectation is given by

$$\mathbb{E}_t^\theta[x_{t+1}] = \rho_x x_t + \theta(\rho_x x_t - \rho_x^2 x_{t-1}) = \rho_x x_t + \theta \rho_x \varepsilon_t$$

- With an i.i.d. shock, no (exogenous) extrapolation:

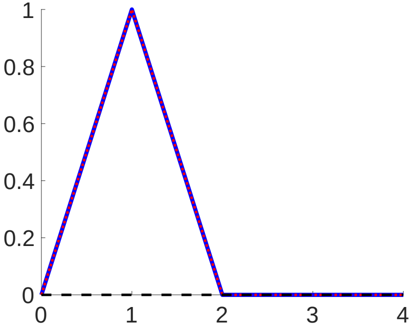
$$\mathbb{E}_t^\theta[x_{t+1}] = \rho_x x_t + \theta(\rho_x x_t - \rho_x^2 x_{t-1}) = \rho_x x_t + 0 \times \rho_x \varepsilon_t = \rho_x x_t$$

- (Endogenous) extrapolations delivered with endogenous state variable
  - Extrapolation triggered even with i.i.d. shocks

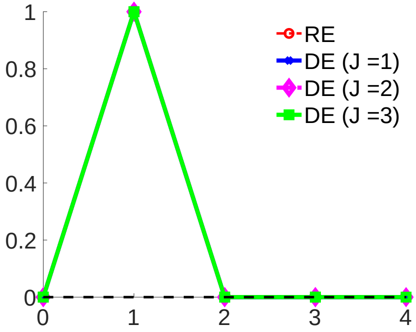
When does DE on endogenous variables matter?

$$y_t = a\,\mathbb{E}_t^{\theta,J} y_{t+1} + c\,y_{t-1} + \epsilon_t; \quad \epsilon_t \sim iid\ N(0,1)$$

Assume $a = 0.5$, $c = 0$



Shock process ϵ_t



Solution for y_t when $c = 0$

When does DE on endogenous variables matter?

$$y_t = a\,\mathbb{E}_t^{\theta,1} y_{t+1} + c\, y_{t-1} + \epsilon_t; \quad \epsilon_t \sim iid\, N(0,1)$$

$$\mathbb{E}_t^{\theta,1} y_{t+1} = (1+\theta)\mathbb{E}_t y_{t+1} - \theta \mathbb{E}_{t-1} y_{t+1}$$

Assume $a = 0.5$, $c = 0.4$, $J = 1$

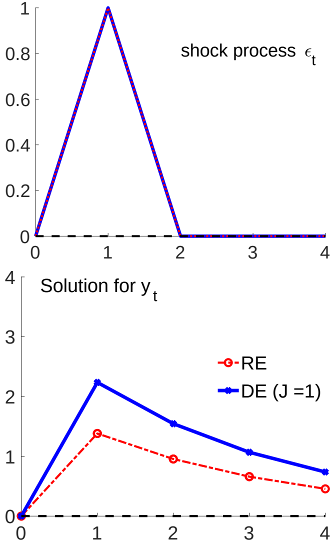
RE ($\theta = 0$):

$$y_t = \phi y_{t-1} + \frac{1}{1 - a\phi}\epsilon_1$$

DE at $J = 1$:

$$y_t = \phi y_{t-1} + \frac{1}{1 - (1+\theta)a\phi}\epsilon_1$$

where $\phi \equiv \frac{1-\sqrt{1-4ac}}{2a}$



General Model

- Exogenous process

$$\mathbf{x}_t = \mathbf{A}\mathbf{x}_{t-1} + \mathbf{v}_t$$

- Recursive model:

$$\mathbb{E}_t^{\theta}[\mathbf{F}\mathbf{y}_{t+1} + \mathbf{G_1}\mathbf{y}_t + \mathbf{M}\mathbf{x}_{t+1} + \mathbf{N_1}\mathbf{x}_t] + \mathbf{G_2}\mathbf{y}_t + \mathbf{H}\mathbf{y}_{t-1} + \mathbf{N_2}\mathbf{x}_t = 0$$

- **Question:** How to compute the equilibrium $\mathbb{E}_t^{\theta}[\mathbf{F}\mathbf{y}_{t+1} + \ldots]$?
  1. Equilibrium $\mathbf{y}_t$?
  2. Combinations of future and contemporaneous variables?

Loglinearization

Existence of Rational Expectations Representation

Proposition (Multivariate RE Representation)

The model admits the following RE representation:

$$
\begin{aligned}
\mathbf{F}\mathbb{E}_t[\mathbf{y}_{t+1}] + \mathbf{G}\mathbf{y}_t + \mathbf{H}\mathbf{y}_{t-1} + \mathbf{M}\mathbb{E}_t[\mathbf{x}_{t+1}] + \mathbf{N}\mathbf{x}_t \\
+\mathbf{F}\theta\big(\mathbb{E}_t[\mathbf{y}_{t+1}] - \mathbb{E}_{t-1}[\mathbf{y}_{t+1}]\big) \\
+\mathbf{M}\theta\big(\mathbb{E}_t[\mathbf{x}_{t+1}] - \mathbb{E}_{t-1}[\mathbf{x}_{t+1}]\big) \\
+\mathbf{G}_1\theta\big(\mathbf{y}_t - \mathbb{E}_{t-1}[\mathbf{y}_t]\big) \\
+\mathbf{N}_1\theta\big(\mathbf{x}_t - \mathbb{E}_{t-1}[\mathbf{x}_t]\big) = 0
\end{aligned}
$$

- We can take standard steps to solve.

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1. Introduction
2. Preliminaries: Overview of Diagnostic Expectations
3. **Theoretical: Endowment economy**
4. Empirical: Quantitative SOE-DSGE models

An endowment economy

- Households:
  1. maximize quadratic utility s.t. the sequential budget constraint:

$$c_t + (1 + r)d_{t-1} = y_t + d_t$$

  2. have access to international bond with exogenous interest rate r

- Rest of the model:
  1. Endowment (income) is exogenously given:

$$y_t = \rho y_{t-1} + \varepsilon_t$$

  2. Trade balance: $tb_t = y_t - c_t$

- Implications:
  - A random walk consumption path
  - Under RE, an initial TB surplus following temporary income shock

Representative Household Problem

- Consider household optimization under DEs:

$$\max_{d_t^\theta} u\left(c_t^\theta\right) + \mathbb{E}_t^\theta\left[v\left(d_t^\theta\right)\right]$$

  - Selective memory recall based on distant past: time-inconsistency
  - Naïveté: from t+1 onwards, the agent assumes future herself operating with RE

$$v\left(d_t^\theta\right) = \max_{d_{t+1}^{RE}} u\left(c_{t+1}^{RE}\right) + \mathbb{E}_{t+1}\left[v\left(d_{t+1}^{RE}\right)\right]$$

- In the PIH model, consumption optimization under DE becomes:

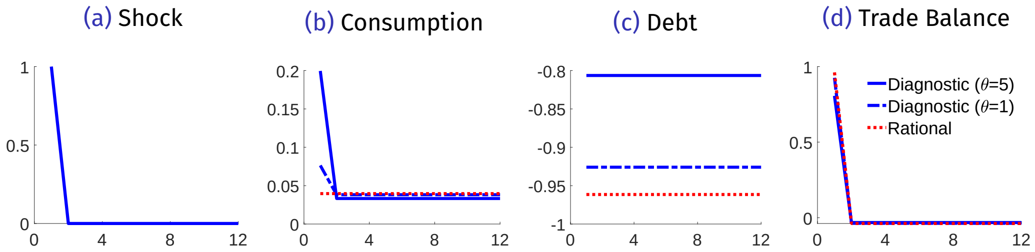
$$c_t^\theta = \mathbb{E}_t^\theta\left[c_{t+1}^{RE}\right]$$

where

$$\mathbb{E}_t^\theta\left[c_{t+1}^{RE}\right] = \mathbb{E}_t\left[c_{t+1}^{RE}\right] + \underbrace{\theta\left(\mathbb{E}_t\left[c_{t+1}^{RE}\right] - \mathbb{E}_{t-J}\left[c_{t+1}^{RE}\right]\right)}_{\text{memory distortion}}$$

Overreaction and endogenous extrapolation

Figure: Implications of an i.i.d. income shock

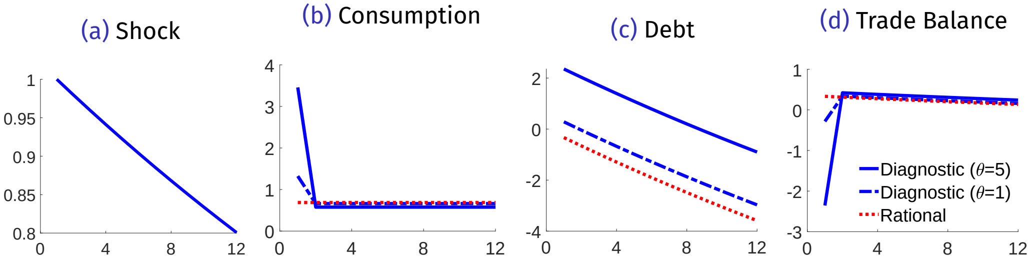


Two predictions from DE: 1) Impact over-reaction

2) Systematic reversals

Countercyclical trade balance under DE

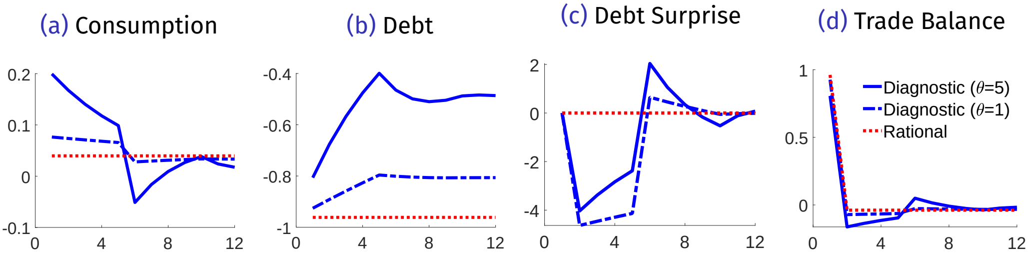
Figure: Implications of a persistent income shock



Note: Impact trade balance deterioration

Endogenous, Repeated Booms and Busts

Figure: Implications of an i.i.d. income shock with distant memory (J=5)



Note: Endogenous booms and busts and trade balance reversals

Outline

Garcia-Cicco, Pancrazi & Uribe (GPU), 2010

- A Quantitative SOE-DSGE Model with multiple shocks and frictions
- Consider household optimization under DE:

$$\max_{\{C_t, h_t, K_{t+1}, D_{t+1}\}} \nu_t U(C_t, X_{t-1} h_t) + \mathbb{E}_t^{\theta} \left[\sum_{s=t+1}^{\infty} \nu_s \beta^{s-t} U(C_s, X_{s-1} h_s) \right]$$

subject to

$$\frac{D_{t+1}}{1+r_t} = D_t - W_t h_t - u_t K_t + C_t + S_t + I_t + \frac{\phi}{2}\left(\frac{K_{t+1}}{K_t} - g\right)^2 K_t - \Pi_t$$

$$K_{t+1} = (1-\delta) K_t + I_t$$

Rest of the Model

- Production

$$Y_t = a_t K_t^{\alpha} (X_t h_t)^{1-\alpha}$$

$$M_t \geq \eta W_t h_t$$

- Interest rate

$$r_t = \bar{r} + \psi p(\tilde{d}_{t+1}) + \mu_t$$

- Implications under RE:
  - Trend shocks not the primary source of excess consumption volatility

Estimation

- Reference distribution is the weighted average of lagged RE expectations (Bianchi, Ilut & Saijo 2023)

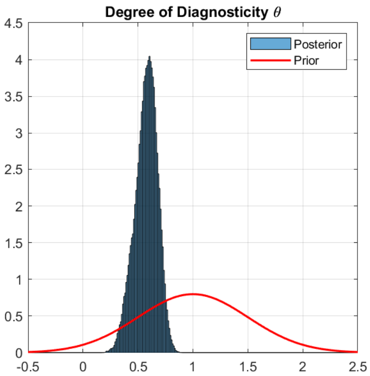
$$\mathbb{E}_t^{\theta}[z_{t+1}^{RE}] = \mathbb{E}_t[z_{t+1}^{RE}] + \theta(\mathbb{E}_t[z_{t+1}^{RE}] - \mathbb{E}_t^{r}[z_{t+1}^{RE}])$$

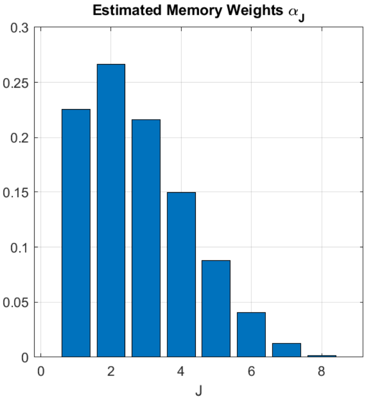
  where

$$\mathbb{E}_t^{r}[z_{t+1}^{RE}] = \sum_{j=1}^{J} \alpha_j \mathbb{E}_{t-j}[z_{t+1}^{RE}]$$

- The same data as in GPU: g^Y, g^C, g^I, and TB/Y for Argentina (1900-2005)
- Structural parameters using Bayesian method
  - (η, ϕ, ψ) + (shock parameters) + DE parameters (θ, α_j)

Estimated DE Parameters





- Estimated $\hat{\theta} \simeq 0.6$ is consistent with existing literature

Parameter	Description	Diagnostic Median	Diagnostic [05, 95]	Rational Median	Rational [05, 95]
η	working capital	0.3127	[0.0342, 1.0606]	0.4669	[0.1696, 0.8540]
ϕ	capital adj.	4.9589	[3.2279, 6.7607]	3.6746	[2.5468, 5.1732]
ψ	IDEIR parameter	0.0617	[0.0177, 0.1685]	0.5690	[0.2730, 1.1124]
persistence of shocks					
ρ_g	TFP trend	0.3637	[0.0791, 0.7680]	0.5140	[0.1216, 0.7850]
ρ_z	TFP temporary	0.7565	[0.6694, 0.8172]	0.8200	[0.7235, 0.8978]
ρ_μ	interest rate	0.5010	[0.2416, 0.6745]	0.8680	[0.7799, 0.9360]
ρ_s	gov. spending	0.3287	[0.0776, 0.7401]	0.3447	[0.0912, 0.7240]
ρ_ν	preference shift	0.5735	[0.4448, 0.6766]	0.7407	[0.6362, 0.8191]
std. of shocks					
σ_g	TFP trend	0.0021	[0.0001, 0.0112]	0.0049	[0.0002, 0.0231]
σ_z	TFP temporary	0.0341	[0.0302, 0.0386]	0.0318	[0.0270, 0.0362]
σ_μ	interest rate	0.0322	[0.0196, 0.0496]	0.0640	[0.0409, 0.0985]
σ_s	gov. spending	0.0034	[0.0002, 0.0244]	0.0038	[0.0002, 0.0273]
σ_ν	preference shift	0.2974	[0.2513, 0.3517]	0.3375	[0.2814, 0.4053]

Model Fits: Second Moments

	g^Y	g^C	g^I	TB/Y
Data (STD $\times$ 100)	5.3	7.5	20.4	5.2
RE	5.9	8.0	18.4	4.8
DE	6.1	8.0	17.9	5.8
Corr w/ g^Y				
Data		0.72	0.67	-0.04
RE		0.78	0.53	-0.14
DE		0.78	0.66	-0.20
Corr w/ TB/Y				
Data		-0.27	-0.19	
RE		-0.36	-0.31	
DE		-0.30	-0.35	
First-order autocorr				
Data	0.11	0.00	0.32	0.58
RE	0.01	-0.06	-0.12	0.52
DE	-0.03	-0.14	0.06	0.70

Variance Decomposition

	g^Y	g^C	g^I	TB/Y
TFP Trend				
RE	0.96	0.52	0.19	0.01
DE	0.26	0.15	0.16	0.63
TFP Temporary				
RE	90.0	46.7	16.6	0.90
DE	94.3	55.8	42.6	21.3
Interest Rate				
RE	5.21	17.7	59.9	85.9
DE	3.22	5.52	44.6	59.7
Preference Shift				
RE	3.47	33.3	17.5	10.4
DE	1.87	36.7	6.59	16.4

- Negligible role for TFP trend shock (in contrast to Aguiar and Gopinath 2007)
- Highlighting the role of temporary TFP shock in fluctuations

Robustness: Misspecified environment

- Consider GPU without preference shocks
  - GPU model already fine-tuned to best explain data under RE
  - Adding DE onto it with same data unlikely to yield significantly different results
- ... RE doesn't work effectively!

Whole Sample

Model Fits: Second Moments

	g^Y	g^C	g^I	TB/Y
STD × 100				
Data	5.3	7.5	20.4	5.2
RE	30.4	30.8	36.6	15.7
DE	9.0	10.7	20.2	18.2
Corr w/ g^Y				
Data		0.72	0.67	-0.04
RE		0.98	0.87	0.84
DE		0.85	0.67	0.37

- Much better fit with DE

Variance Decomposition

	g^Y	g^C	g^I	TB/Y
TFP Trend				
RE	95.5	95.4	64.8	90.8
DE	48.2	62.7	14.1	92.8
TFP Temporary				
RE	3.70	2.29	7.44	0.17
DE	49.7	32.6	36.4	0.74
Interest Rate				
RE	0.72	2.19	25.9	8.90
DE	1.60	4.23	44.4	6.19

- Trend shocks dominate with RE
- Temp. shocks important with DE

Post-1945 sample

Endogeneous demand shifter in DE

- Linearized consumption Euler equation under RE:

$$\hat{\lambda}_t = \hat{R}_t - \gamma \hat{g}_t + \mathbb{E}_t \hat{\lambda}_{t+1}, \qquad \lambda_t = \nu_t \left(c_t - \frac{h_t^{\omega}}{\omega} \right)^{-\gamma}$$

- Under DE:

$$\hat{\lambda}_t^{\theta} = \hat{R}_t^{\theta} - \gamma \hat{g}_t^{\theta} + \mathbb{E}_t^{\theta} \hat{\lambda}_{t+1}^{RE} - \theta \gamma \hat{g}_{J,t}^{*}$$

$$\hat{g}_{J,t}^{*} \equiv \sum_{j=1}^{J} \hat{g}_{t-j+1} - \sum_{k=1}^{J} \sum_{j=1}^{J} \alpha_j \mathbb{E}_{t-j} [\hat{g}_{t-k+1}^{RE}]$$

- $\hat{g}_{J,t}^{*}$ acts as an endogenous demand shifter (as opposed to ν_t under RE)

Summary

- Incorporating DE into a dynamic open macro model
- Rich theoretical/empirical insights in the context of open economy models

Diagnostic PDF with distant memory

- Diagnostic pdf (with distant memory) is defined as

$$f_t^{\theta,J}(x_{t+1}) = \underbrace{f(x_{t+1}|G_t)}_{\text{true pdf}} \cdot \underbrace{\left[\frac{f(x_{t+1}|G_t)}{f(x_{t+1}|G_{t-1})}\right]^{\theta\alpha_{1,J}} \cdots \left[\frac{f(x_{t+1}|G_t)}{f(x_{t+1}|G_{t-J})}\right]^{\theta\alpha_{J,J}}}_{\text{distortion}} \cdot C$$

where $\theta\alpha_{1,J} + \cdots + \theta\alpha_{J,J} = \theta$

- Information sets:
  - G_t: current state t
  - G_{t-j}: reference state with information available at $t-j$.

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Loglinearization Matters...

Consider household optimization under diagnostic expectations (J=2):

$$\max_{\{C_t, D_{t+1}\}} \log C_t + \mathbb{E}_t^\theta \left[\sum_{s=t+1}^{\infty} \beta^{s-t} \log(C_s) \right]$$

subject to a resource constraint:

$$C_t + \frac{B_{t+1}}{(1+i_t)} = Y_t + B_t, \qquad Y_t = A_t, \qquad \log A_t = \log A_{t-1} + \varepsilon_t$$

First-order condition:

$$\left(\frac{A_t}{C_t}\right)\left(\frac{A_{t-1}}{A_t}\right)\left(\frac{A_{t-2}}{A_{t-1}}\right) = \beta(1+i_t)\mathbb{E}_t^\theta \left[\left(\frac{A_{t+1}}{C_{t+1}}\right)\left(\frac{A_t}{A_{t+1}}\right)\left(\frac{A_{t-1}}{A_t}\right)\left(\frac{A_{t-2}}{A_{t-1}}\right) \right]$$

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Post-1945

Model Fits: Second Moments

	g^Y	g^C	g^I	TB/Y
STD × 100				
Data	5.1	6.7	17.2	3.3
RE	11.8	12.4	20.0	4.8
DE	6.3	7.7	20.7	2.8
Corr w/ g^Y				
Data		0.90	0.87	-0.21
RE		0.96	0.78	0.45
DE		0.93	0.90	-0.54

Variance Decomposition

	g^Y	g^C	g^I	TB/Y
TFP Trend				
RE	80.7	79.2	26.8	66.7
DE	16.0	19.8	0.20	0.91
TFP Temporary				
RE	13.4	8.06	17.2	0.31
DE	52.8	21.6	48.1	5.77
Interest Rate				
RE	5.75	12.3	52.3	32.7
DE	30.9	56.8	49.0	92.6

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